Inhomogeneous pressure cosmology as a complement of inhomogeneous density (LTB) cosmology

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1. Introduction. Do we live in the center of the Universe?

- Einstein equations are complicated and to solve them we just assume symmetries (Occam's razor - if we play with simple symmetric models observationally, we do not need to bother about any more complicated ones).
- Why not to paradigm this by a fundamental principle - the Copernican Principle that we do not live in the center of the Universe (we really do not want to be special in the Universe).
- Observations we made are from one point in the Universe and extend only onto the past light cone.
- Even CMB we observe from one point - this proves isotropy, but not necessarily homogeneity (isotropy with respect to any point in the Universe).
- Should not we first start from observations of the cone and then make conclusions related to modeling the universe (cf. observational cosmology programme of G.F.R. Ellis and collaborators).

Center of the Universe?

- In other words - homogeneity needs a check.
- Suppose we have an inhomogeneous model of the Universe with the same (small) number of parameters as a homogeneous dark energy model and they both fit observations very well.
- Could we differentiate between these two models (some tests have been proposed recently but they are not very efficient - see later).
- The simplest models of this type are the models with only one or two parameters which are spherically symmetric (isotropic with respect to only one point).
- Some even argue (Clarkson and Barrett, CQG ’99, ’00) that even if we proved isotropy for all observers in the universe, it would still not be enough to prove homogeneity, unless we proved all the fluid components in the universe were comoving perfect fluids.

Center of the Universe?

- There are only two complementary models of the spherically symmetric Universe which can be tested against homogeneous dark energy models.
- These are the inhomogeneous density (dust shells) Lemaître-Tolman-Bondi (LTB) models and inhomogeneous pressure (gradient of pressure shells) Stephani models.
- Apparently, most of the researchers for some reasons investigate the former and only few investigate the latter, though there is no special reason to do so.
- MPD and Hendry (Ap.J. '98) first compared inhomogeneous models (no matter if density or pressure) of the Universe with observational data from supernovae and showed that they can be fitted.
- Despite inhomogeneous density (LTB) models were theoretically explored before (since Lemaître - 1933) only later they were tested observationally (e.g. K. Tomita, prog. Theor. Phys. **106**, 929 (2001); K. Bolejko, astro-ph/0512103).

2. Complementary models of the spherically symmetric Universe

I will discuss the advantages of the inhomogeneous pressure models and show that they also may fit observations, so that they are a good candidate for explanation of cosmic acceleration by an inhomogeneity.

In order to make a complementary analysis with LTB models the following table proves useful:

	pressure	density
FRW	$p = p(t)$	$\varrho = \varrho(t)$
LTB	$p = p(t)$	$\varrho = \varrho(t, r)$ - nonuniform
Stephani	$p = p(t, r)$ - nonuniform	$\varrho = \varrho(t)$

SS Lemaître-Tolman-Bondi Universe

– is the only spherically symmetric solution of Einstein equations for pressureless matter ($T^{ab} = \varrho u^a u^b$) and no cosmological term (G. Lemaître, Ann. Soc. Sci. Brux. **A 53**, 51 (1933); R.C. Tolman, Proc. Natl. Acad. Sci., **20**, 169 (1934); H. Bondi MNRAS **107**, 410 (1947))

$$ds^2 = -dt^2 + \frac{R'^2}{1-K} dr^2 + R^2 (d\theta^2 + sin^2\theta d\phi^2) \quad , \tag{1}$$

where

$$R = R(t,r); \qquad R' = \partial R / \partial r; \qquad K = K(r) \quad . \tag{2}$$

The Einstein equations reduce to

$$\dot{R}^2 = \frac{2M(r)}{R} - K(r); \qquad 2M' = \kappa \varrho R^2 R' \quad , \tag{3}$$

SS Lemaître-Tolman-Bondi Universe

and are solved by

$$R(r,\eta) = \frac{M(r)}{K(r)}\Phi'(\eta); \qquad t(r,\eta) = T_0(r) + \frac{M(r)}{K^{3/2}(r)}\phi'(\eta) \ , \tag{4}$$

where for $K(r) < 0$ (hyperbolic), $K(r) = 0$ (parabolic), and $K(r) > 0$ (elliptic) we have appropriately (K may change sign even in one regular domain!)

$$\Phi(\eta) = (\sinh\eta - \eta; \eta^3/6; \eta - \sin\eta) \ . \tag{5}$$

Regularity conditions:
- existence of a regular center of symmetry $r = 0$ – implies $R(t,0) = \dot{R}(t,0) = 0$ and $M(0) = M'(0) = K(0) = K'(0) = 0$ and $R' \to 1$.
- hypersurfaces of constant time are orthogonal to 4-velocity and are of topology S^3 – implies the existence of a second center of symmetry $r = r_c$ (with some 'turning value' $0 < r_{tv} < r_c$)
- a 'shell-crossing' singularity should be avoided – implies $R'(t,r) \neq 0$ except at turning values

SS Lemaître-Tolman-Bondi Universe

Kinematic characteristics of the model:

$$u_{a;b} = \frac{1}{3}\Theta h_{ab} + \sigma_{ab} \quad , \tag{6}$$

Expansion scalar:

$$\Theta = \frac{2\dot{R}}{R} + \frac{\dot{R}'}{R'} \, , \tag{7}$$

Shear tensor and scalar:

$$\sigma^{ab} = \Sigma\zeta^{ab}; \qquad \zeta^{ab} \equiv h^{ab} - 3v^a v^b; \tag{8}$$

$$\Sigma = \frac{1}{6}\sigma_{ab}\zeta^{ab} = -\frac{1}{3}\left(\frac{\dot{R}'}{R'} - \frac{\dot{R}}{R}\right) \quad , \tag{9}$$

and $v^a = \sqrt{h^{rr}}\delta^a_r$ is the unit vector orthogonal to u^a and to the 2-spheres orbits of $SO(3)$.

living in a "void" against living in an "exotic star"

Friedmann limit is obtained for:

$$R(t,r) = a(t)r; \qquad M(r) = M_0 r^3; \qquad K(r) = k_0 r^2 \, , \tag{10}$$

In the context of dark energy problem there has been more interest in LTB models since it could explain the acceleration only due to inhomogeneity - a claim is that

we are living in a spherically symmetric void of density

J. Uzan, R. Clarkson, G.F.R. Ellis (PRL, **100**, 191303 (2008))
R.R. Caldwell and A. Stebbins (PRL, **100**, 191302 (2008))
C. Clarkson, B. Bassett and T. H-Ch. Lu (PRL, **101**, 011301 (2008))
R.A. Sussmann, 0807.1145
K. Bolejko, 0807.2891

I would then suggest that

we are living in a spherically symmetric evolving “exotic star” of variable pressure

SS Stephani Universe

– is the only spherically symmetric solution of Einstein equations for perfect-fluid energy-momentum tensor ($T^{ab} = (\varrho + p)u^a u^b + pg^{ab}$) which is conformally flat and embeddable in a 5-dimensional flat space (H. Stephani Commun. Math. Phys. **4**, 167 (1967); A. Krasiński, GRG **15**, 673 (1983)). After introducing a Friedmann-like time coordinate (cf. later) we have

$$ds^2 = -\frac{a^2}{\dot{a}^2}\frac{a^2}{V^2}\left[\left(\frac{V}{a}\right)^{\cdot}\right]^2 dt^2 + \frac{a^2}{V^2}\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] , \quad (11)$$

where

$$V(t,r) = 1 + \frac{1}{4}k(t)r^2 , \quad (12)$$

and $(\ldots)^{\cdot} \equiv \partial/\partial t$. The function $a(t)$ plays the role of a generalized scale factor, $k(t)$ has the meaning of a time-dependent "curvature index", and r is the radial coordinate.

SS Stephani Universe

The energy density and pressure are given by

$$\varrho(t) = 3C^2(t) \equiv 3\left[\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k(t)}{a^2(t)}\right] , \tag{13}$$

$$p(t,r) = -3C^2(t) + 2C(t)\dot{C}(t)\frac{\left[\frac{V(t,r)}{a(t)}\right]}{\left[\frac{V(t,r)}{a(t)}\right]^{\cdot}} , \tag{14}$$

and generalize the standard Einstein-Friedmann relations

$$\varrho = 3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) , \tag{15}$$

$$p = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) \tag{16}$$

to inhomogeneous models.

SS Stephani Universe

Kinematic characteristic of the model:

$$u_{a;b} = \frac{1}{3}\Theta h_{ab} - \dot{u}_a u_b \ , \qquad \dot{u} \equiv (\dot{u}_a \dot{u}^a)^{\frac{1}{2}} \ . \tag{17}$$

where u is the acceleration scalar and the acceleration vector

$$\dot{u}_r = \frac{\left\{ \frac{a^2}{\dot{a}^2} \frac{a^2}{V^2} \left[\left(\frac{V}{a} \right)^{\cdot} \right] \right\}_{,r}}{\frac{a^2}{\dot{a}^2} \frac{a^2}{V^2} \left[\left(\frac{V}{a} \right)^{\cdot} \right]} \tag{18}$$

while the expansion scalar is the same as in FRW model, i.e.,

$$\Theta = 3\frac{\dot{a}}{a} \ . \tag{19}$$

3. Inhomogeneous pressure models - properties.

Properties of general Stephani models:

- really inhomogeneous (not even SS) - they do not admit any spacetime symmetry at all
- the 3-dimensional hyperspaces of constant time are maximally symmetric
- the models are conformally flat (Weyl tensor $C_{abcd} = 0$)
- can be embedded into a 5-dimensional flat pseudoeuclidean space (they are embedding class one – in general any 4-dim manifold can be embedded at least locally in a 10-dim flat space)
- matter does not move along geodesics (there is non-zero acceleration $\dot{u}_a \neq 0$); models are shearfree $\sigma_{ab} = 0$
- the curvature index $k = k(t)$ changes in time so that the spatial curvature may change during evolution
- possess the Friedmann limit when the curvature index $k(t) \to$ const. $= 0, \pm 1$

Inhomogeneous pressure models - properties

The general metric reads as

$$ds^2 = -\frac{a^2}{\dot{a}^2}\frac{a^2}{V^2}\left[\left(\frac{V}{a}\right)^{\cdot}\right]^2 dt^2 + \frac{a^2}{V^2}\left[dx^2 + dy^2 + dz^2\right], \quad (20)$$

$$V(t,x,y,z) = 1 + \frac{1}{4}k(t)\left\{[x - x_0(t)]^2 + [y - y_0(t)]^2 + [z - z_0(t)]^2\right\},$$

and x_0, y_0, z_0 are arbitrary functions of time. This is just a generalization of the FRW metric in isotropic coordinates

$$ds^2 = -dt^2 + \frac{a^2(t)}{1 + \frac{1}{4}kr^2}\left(dr^2 + r^2 d\Omega^2\right); \qquad r^2 = x^2 + y^2 + z^2 \quad (21)$$

which by a transformation $\bar{r} = 1 + (1/2)kr^2$ can be brought to a standard form

$$d\bar{s}^2 = -dt^2 + a^2(t)\left(\frac{d\bar{r}^2}{1 - k\bar{r}^2} + \bar{r}^2 d\Omega^2\right). \quad (22)$$

Inhomogeneous pressure models - topology

Topology can be uncovered, if we assume the energy density to be constant, i.e.,

$$\frac{8\pi G}{c^2}\varrho = 3C_0^2 = \text{const.}\ , \tag{23}$$

$$\frac{8\pi G}{c^4}p = -3C_0^2 = \text{const.}\ , \tag{24}$$

which is essentially the de Sitter Universe with dark energy equation of state ($w = -1$) with global topology being $S^3 \times R$ represented by a one-sheet hyperboloid,

but with local topology of the constant time hypersurfaces (index $k(t)$) changing in time.

Usually we cut hyperboloid by either $k = 1$ (S^3 topology), $k = 0$ (R^3) or $k = -1$ (H^3).

Here we have "3-in-1" and the Universe may either

"open up" or "close down".

Inhomogeneous pressure models - topology

General model:

- Global topology still $S^3 \times R$. However, they are just specific deformations of the de Sitter hyperboloid near the "neck circle".
- The center of symmetry is moving around the deformed hyperboloid.
- In fact, due to a choice of the radial coordinate, there are two antipodal centers of symmetry

Inhomogeneous pressure models - singularities, EOS

- standard Big-Bang singularities $a \to 0$, $\varrho \to 0$, $p \to 0$ are possible (FRW limit)
- Finite Density (FD) singularities of pressure appear at some particular values of the spatial coordinates x, y, z (or a radial coordinate r, if in a SS model)
- Π-boundary - a spacelike boundary which divides each negative curvature $k(t) < 0$ section onto the two sheets (the "far sheet" and the "near-sheet")
- Π-boundary appears whenever $V(t,r) = 1 + (1/4)k(t)[(x - x_0)^2 + \ldots] = 0$
- the Universe behaves asymptotically de Sitter on a Π-boundary ($p = -\varrho$)
- There is no global equation of state - it changes from place to place (depends on x, y, z or r) and on the hypersurfaces $t = \text{const}$.

Exact inhomogeneous pressure models

I found two explicit models which are called Model I and Model II (note: time coordinate will be labeled τ instead of t and the scale factor $R(t)$ instead of $a(t)$). For the Model I we have

$$k(\tau) = -4\frac{a}{c^2}R(\tau), \tag{25}$$

$$R(\tau) = a\tau^2 + b\tau + d, \tag{26}$$

$$V(\tau, r) = 1 - \frac{a}{c^2}\left(a\tau^2 + b\tau + d\right) r^2, \tag{27}$$

$$\Delta \equiv 4ad - b^2 + 1 = 0, \tag{28}$$

with a, b, d = const. and for the cosmic time τ taken in sMpc/km we have: [a] = $km^2/(s^2 Mpc)$, [b] = km/s and [c] = Mpc.

Exact inhomogeneous pressure models

For the Model II we have

$$k(\tau) = -\frac{\alpha\beta}{c^2}R(\tau)\ , \tag{29}$$

$$R(\tau) = \beta\tau^{\frac{2}{3}}\ , \tag{30}$$

$$V(\tau, r) = 1 - \frac{1}{4c^2}\alpha\beta^2\tau^{\frac{2}{3}}r^2\ , \tag{31}$$

with α, β = const. with $[\alpha]$ = $(s/km)^{\frac{2}{3}}Mpc^{-\frac{4}{3}}$ and $[\beta]$ = $(km/s)^{\frac{2}{3}}Mpc^{\frac{1}{3}}$. Both models possess the Friedman limit; ($a \to 0$ for MI and $\alpha \to 0$ for MII). The common point between MI and MII is that for them $\left(\frac{k}{R}\right)_{,\tau} = 0$, where $(\ldots)_{,\tau} \equiv \frac{\partial}{\partial\tau}$.

Inhomogeneous pressure models - null geodesics

The four-velocity and the acceleration for MI and MII are

$$u_\tau = -\, c\frac{1}{V}\ , \qquad \dot{u}_r = -\, c\frac{V_{,r}}{V}\ . \tag{32}$$

The components of the vector tangent to zero geodesic are

$$k^\tau = \frac{V^2}{R}\ , \quad k^r = \pm\frac{V^2}{R^2}\sqrt{1 - \frac{h^2}{r^2}}\ , \quad k^\theta = 0\ , \quad k^\varphi = h\frac{V^2}{R^2 r^2}\ , \tag{33}$$

where h = const., and the plus sign in applies to a ray moving away from the centre, while the minus sign applies to a ray moving towards the centre. The acceleration scalar for MI and MII, respectively, is

$$\dot{u} \equiv (\dot{u}_a \dot{u}^a)^{\frac{1}{2}} = \frac{V_{,r}}{R} = \begin{cases} -\, 2\frac{a}{c^2}r, \\ -\, \frac{1}{2}\alpha\beta r, \end{cases} \tag{34}$$

and it does not depend on the time coordinate at all.

Inhomogeneous pressure models - redshift

The point:
The further away from the center $r = 0$ is an observer, the larger acceleration he subjects.

The redshift is given by (for MI and MII, respectively)

$$1 + z = \frac{(u_a k^a)_O}{(u_a k^a)_G} = \begin{cases} \dfrac{\left[\dfrac{1 - \frac{a}{c^2}(a\tau^2 + b\tau + d)r^2}{a\tau^2 + b\tau + d}\right]_O}{\left[\dfrac{1 - \frac{a}{c^2}(a\tau^2 + b\tau + d)r^2}{a\tau^2 + b\tau + d}\right]_G}, \\ \dfrac{\left[\dfrac{1 - \frac{1}{4}\alpha\beta^2\tau^{\frac{2}{3}}r^2}{\beta\tau^{\frac{2}{3}}}\right]_O}{\left[\dfrac{1 - \frac{1}{4}\alpha\beta^2\tau^{\frac{2}{3}}r^2}{\beta\tau^{\frac{2}{3}}}\right]_G}. \end{cases} \tag{35}$$

4. Inhomogeneous pressure models - observations.

Taking complexity of models into account, it is best to apply the series expansion of the redshift-magnitude formula (Kristian and Sachs 1966) given by (calculated to higher-orders in MPD & Stachowiak '06)

$$m_{bol} = M - 5\log_{10}\left(u_{a;b}K^aK^b\right)_O + 5\log_{10}cz + \frac{5}{2}\left(\log_{10}e\right)\left\{\left(4 - \frac{u_{a;bc}K^aK^bK^c}{\left(u_{a;b}K^aK^b\right)^2}\right)z + \mathbf{O}\left(z^2\right)\right\}_O , \tag{36}$$

where

$$u_{a;b} = \frac{1}{3}\Theta h_{ab} - \dot{u}_a u_b\ , \quad h_{ab} \equiv g_{ab} + u_a u_b\ , \quad K^a \equiv \frac{k^a}{u_b k^b}\ , \quad u_a u^a = -1\ . \tag{37}$$

The projection of K^a onto the spatial hypersurfaces ortogonal to u_a is a spatial unit vector pointing in the observer direction

$$n^a = -u^a - K^a, \quad n^a n_a = 1\ . \tag{38}$$

Inhomogeneous pressure models - MI central observations

After some algebra we find redshift-magnitude relations for the Model I as follows (since the only inhomogeneous pressure parameter is a, then we have chosen $b = 1$ and $d = 0$ without loosing the generality)

$$m = M + 25 + 5\log_{10}\left[cz\left(\frac{a\tau_0^2 + \tau_0}{2a\tau_0 + 1}\right)\right] + 1.086\left[1 + 4a\frac{\left(a\tau_0^2 + \tau_0\right)}{\left(2a\tau + 1\right)^2}\right]z. \quad (39)$$

This relation has no difference with the FRW relation

$$m_{bol} = M - 5\log_{10}\tilde{H}_0 + 5\log_{10}cz + 1.086\,(1 - \tilde{q}_0)\,z, \quad (40)$$

provided we have defined the Hubble and deceleration parameters as

$$\tilde{H}_0 = \frac{2a\tau_0 + 1}{a\tau_0^2 + \tau_0}, \qquad \tilde{q}_0 = -4a\frac{a\tau_0^2 + \tau_0}{(2a\tau_0 + 1)^2}, \quad (41)$$

and they may be taken with the same values as in FRW models.

Inhomogeneous pressure models - MII central observations.

For the Model II we have

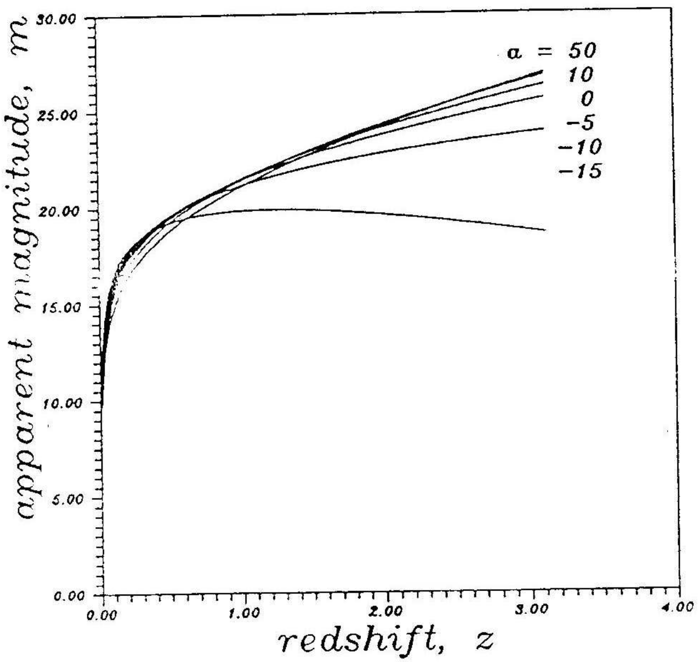
$$m_{bol} = M - 5\log_{10}\tilde{H}_0 + 5\log_{10} cz + 1.086\,(1-\tilde{q}_0)\,z, \tag{42}$$

where

$$\tilde{H}_0 = \frac{2}{3\tau_0}, \qquad \tilde{q}_0 = \frac{1}{2} - \frac{9}{8}c^2\alpha\tau_0^{\frac{4}{3}}. \tag{43}$$

are the Friedman values of the Hubble and the deceleration parameters.

Inhomogeneous pressure - theoretical plots



A plot of the redshift-magnitude relation for the model MI given by the formulas. We have chosen a = -15, -10, -5, 0, 10, 50 km^2/s^2Mpc, $b = 1km/s$, d = 0 with $\tau_0 = 0.02\ sMpc/km$ and $M = -23.5$. The effect of inhomogeneous pressure is similar to the effect of spatial curvature/dark energy (expressed in terms of the deceleration parameter q_0) in FRW models.

Inhomogeneous pressure models against supernovae data

In (MPD + Hendry ’98) we first compared with Perlmutter P97 (Ap.J. 483, 565 (1997)) data which was in favour of deceleration $(a < 0)$, but the advantage was that inhomogeneous pressure models gave a longer age of the Universe.

According to the current SnIa data (77 supernovae of Riess eta al. for z <0.5) we have the best fit values of inhomogeneity parameter a of the Model I to be

$$a = 3645 km^2/s^2 Mpc > 0. \tag{44}$$

Godłowski, Stelmach and Szydłowski (astro-ph/0403534) checked yet another model of the type II which has approximate dust equation of state at the center of symmetry. Their result show that it fits supernovae, Doppler peaks and BBN.

Inhomogeneous pressure - accelerated away observers

Since the acceleration scalar is

$$\dot{u} = -2\frac{a}{c^2}r \ , \tag{45}$$

with r being the radial coordinate of the model, then

the high pressure region is at $r = 0$ (center of symmetry), while the low (negative) pressure regions are outside the center, so that the particles are

accelerated away from the center

which is a similar effect to that caused by the positive cosmological constant in ΛCDM model.

The difference is that in ΛCDM the pressure is constant everywhere while in Stephani models it depends on the spatial coordinates.

Just very recent stuff: Vanderveld, Flanagan, Wassermann (0904.4319) argue that acceleration at the center of symmetry of LTB models can only appear if the model is not smooth at the center - here we do not have such a problem at all!!! Acceleration appears naturally and everything is smooth at the center.

Inhomogeneous pressure models - MII non-central observations

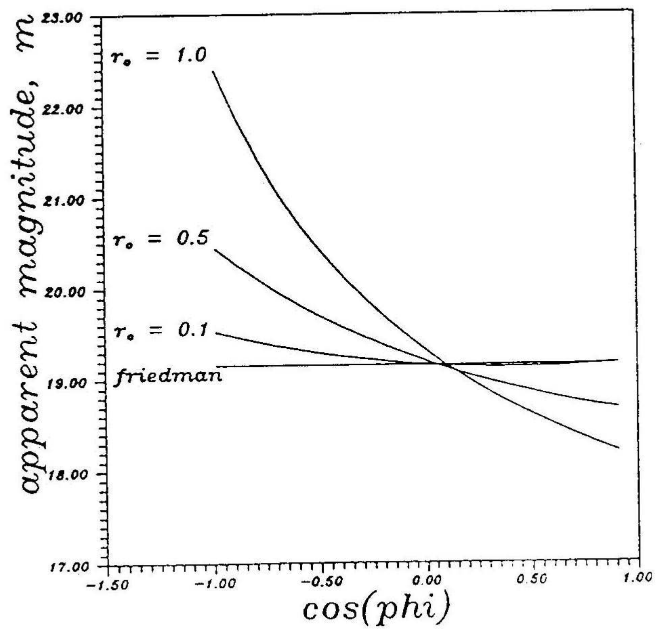
It is most challenge to actually compare inhomogeneous pressure model with supernovae data for non-centrally placed observers, i.e., us being slightly away from the center of symmetry (MPD et al. 2009 - in progress). It emerges that it is even difficult to find a supernovae sample which would contain their directions in the sky (cf. M. Kowalski - private communication). For the Stephani Model II the redshift-magnitude relation in series expansion gets a bit complicated (here $r \neq 0$, and $h \neq 0$ in the previous formulas)

$$m_{bol} = M + 25 + 5\log_{10}\left[\frac{cz}{\frac{2}{3}\frac{1}{\tau_0} + \frac{1}{2}c\alpha\beta r_0\cos\phi}\right] + 1.086z \times$$

$$\left[\frac{\frac{2}{9}\frac{1}{\tau_0^2}\left(1 + \frac{3}{4}\alpha\beta^2\tau_0^{\frac{2}{3}}r_0^2\right) - c\alpha\beta\frac{r_0}{\tau_0}\cos\phi + \frac{1}{2}c^2\alpha\tau_0^{-\frac{2}{3}}\left(1 - \frac{5}{4}\alpha\beta^2\tau_0^{\frac{2}{3}}r_0^2\right)\cos^2\phi}{\left(\frac{2}{3}\frac{1}{\tau_0} + \frac{1}{2}c\alpha\beta r_0\cos\phi\right)^2}\right].$$

Inhomogeneous pressure (observations) - non-centrally placed observers

Here we are an example theoretical plot for non-centrally placed observers



A plot of the dependence of the apparent magnitude on the direction in the sky for the model MII. We fix the redshift of a galaxy to be $z = 0.5$ and $\alpha c^2 = 100(km/sMpc)^{-\frac{4}{3}}, \beta = 1.1 \cdot 10^5 (km/s)^{\frac{2}{3}} Mpc^{\frac{1}{3}}, \tau_0^{-1} = 75 km/(sMpc), -1 < \cos\phi < 1$ and $r_0 = 0.1, 0.5, 1.0$. If $\cos\phi = -1$ galaxies are just behind the center of symmetry with respect to the observer and the apparent magnitude is small. If $\cos\phi = 1$ galaxies are in front of the center of symmetry and the apparent magnitude is large.

5. Inhomogeneous pressure models with sudden future singularities and as interiors of "exotic" stars.

In standard FRW cosmology there exist exotic singularities of pressure with finite scale factor and energy density, i.e.,

$$a = \text{const.}, \quad \dot{a} = \text{const}, \quad \varrho = \text{const}, \quad \ddot{a} \to \pm\infty, \quad p \to \mp\infty \tag{46}$$

called Sudden Future Singularities (SFS).
They appear for the scale factor:

$$a(t) = a_s \left[1 + (1-\delta)\, y^m - \delta\, (1-y)^n\right] \,, \qquad y \equiv \frac{t}{t_s} \tag{47}$$

where $a_s \equiv a(t_s) = \text{const.}$ and $\delta, m, n = \text{const.}$, $1 < n < 2$, and for $t = 0$ one has a Big-Bang singularity.

FD singularities versus SFS singularities

- In inhomogeneous pressure models there are Finite Density singularities of pressure.
- In FRW models there are Sudden Future Singularities of pressure.
- However, they are different: FD singularities are spatial (appear somewhere in space) while SFS are temporal (appear in time on one $(t = t_s)$ of the hypersurfaces).
- The question is if SFS may appear in inhomogeneous pressure models, too?
- I have shown (MPD, PLB '05) that this is the case.
- Later, it was also shown by Barrow and Tsagas (CQG **22**, 1563 (2005)) that SFS are possible in Bianchi types homogeneous universes.

FD singularities versus SFS singularities

Such "inhomogeneized" SFS may appear in a general (no symmetry at all) inhomogeneous pressure model which can be shown by inserting the time derivative of the Stephani energy density function and the function $V(t,x,y,z)$ into the expression for the pressure, i.e.,

$$p(t,x,y,z) = -3\frac{\dot{a}^2}{a^2} - 3\frac{k}{a^2} + \frac{\dot{a}}{a}\left[2\frac{\ddot{a}}{a} - 2\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\left(\dot{k}\frac{a}{\dot{a}} - 2k\right)\right]\frac{\left[\frac{V(t,x,y,z)}{a(t)}\right]}{\left[\frac{V(t,x,y,z)}{a(t)}\right]^{\cdot}} . \tag{48}$$

Then, it is easy to notice that SFS $p \to \pm\infty$ appears with (47) for $\ddot{a} \to -\infty$, if $(V/a)/(V/a)^{\cdot}$ is regular and the sign of the pressure depends on the signs of both $\dot{a}/a$ and $(V/a)/(V/a)^{\cdot}$.

In fact, SF singularities appear independently of FD singularities whenever $\ddot{a} \to -\infty$ and the blow-up of p is guaranteed by the involvement of the time derivative of the function $C(t)$ in (14).

TOV equation for exotic stars

SFS (also called Big-Brake) $a = a_b$ = const., $\dot{a} = 0$ and $\varrho \longrightarrow 0$, $p \longrightarrow \infty$ can be obtained in FRW models filled with anti-Chaplygin gas

$$p = \frac{A^2}{\varrho} \tag{49}$$

with $A =$ const. (Gorini, Kamenschchik et al. PRD 69 (2004), 123512).

In yet another paper Gorini, Kamenschchik et al. (0807.2740) found that Chaplygin gas

$$p = -\frac{A^2}{\varrho} \tag{50}$$

may serve a source for stable exotic star configurations (including phantom) which fulfil appropriate Tolman-Oppenheimer-Volkoff equilibrium equations.

Inhomogeneous pressure model as an interior of an exotic star?

- They also find that there exists a static spherically symmetric configuration in which the central pressure at $r = 0$ is constant while on some shell of constant radius r_s it becomes minus infinity.
- This is an analogue of the FD singularity of Stephani models (though there is no evolution here).
- Besides, on any shell between $r = 0$ and $r = r_s$, there is lower pressure than in the center, so that the particles can just be accelerated away from the center - similar effect as in the Stephani model.
- Of course, for full analogy one needs expansion which is absent here.

6. Conclusions

- Observations from one point in the Universe suggest its isotropy, but not necessarily homogeneity. This gives motivation for studying spherically symmetric models of the Universe.
- Two specific models have been proposed: the Lemaître-Tolman-Bondi model (inhomogeneous density) and the Stephani model (inhomogeneous pressure).
- These models have been preliminary checked against astronomical data which shows that the inhomogeneities may play the role of the dark energy (drive acceleration).
- It opens the question whether we really live in a homogeneous and isotropic (FRW) universe or just in an isotropic (spherically symmetric) universe (at least in a void or an interior of an "exotic star"). Especially, it is interesting to check data for non-centrally placed observers.

conclusions contd.

- The admission of spherical symmetry would be violating the Copernican Principle.
- Inhomogeneous pressure models have another advantage - they can even model total spacetime inhomogeneity.
- Another advantage is that they admit cosmic acceleration in a natural way since pressure is usually related to a repulsive force needed for antigravitation. They also have an analogue with stable exotic star (Chaplygin gas source) configurations.
- An inhomogeneous model with both spatial and temporal pressure singularities can be constructed.