Higher-order brane gravity models

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1. Fourth-order gravities.

When one considers the general gravity theories (e.g. Clifton & Barrow '05; Nojiri & Odintsov '05):

$$S = \chi^{-1} \int d^D \sqrt{-g} f(X, Y, Z) \tag{1}$$

in a D-dimensional spacetime, where X, Y, Z are curvature invariants

$$X = R, \quad Y = R_{ab}R^{ab}, \quad Z = R_{abcd}R^{abcd}, \tag{2}$$

then one immediately faces the 4th order field equations, except when they reduce to the theories with Euler densities of the nth order $I^{(n)}$ (Lovelock '71)

$$S = \int_M d^D x \sqrt{-g} \sum_n \kappa_n I^{(n)} \ , \tag{3}$$

of which examples are just the cosmological constant

$$I^{(0)} = 1, \quad \kappa_0 = -2\Lambda(2\kappa^2)^{-1} = -2\Lambda/16\pi G, \tag{4}$$

the Ricci scalar

$$I^{(1)} = R, \quad \kappa_1 = (2\kappa^2)^{-1}, \tag{5}$$

and the Gauss-Bonnet density

$$I^{(2)} = R_{GB} = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2 \ , \tag{6}$$

($\kappa_2 = \alpha(2\kappa^2)^{-1}$, $\alpha =$ const.) etc.

However, the theories based on the Lagrangians which are the functions of the Euler densities such as

$$f(R) = f(X), \qquad\qquad f(R_{GB}) = f(Z - 4Y + X^2) \; , \tag{7}$$

are again fourth-order.

The fact that the general $f(X, Y, Z)$ theories are the 4th order may have some advantageous consequences onto their observational verification.

2. Formulation of 4th order gravities on the brane - Israel formalism.

In the context of recent interest in string/M-theory it is interesting to formulate and check observationally the general gravity theories within the framework of the brane models (MPD & Balcerzak '08)

$$S = \chi^{-1}\int_M d^D x\sqrt{-g}f(X,Y,Z) + S_{brane} + S_m \ , \tag{8}$$

with the total energy-momentum tensor to be

$$T_a^{\ b} = T_a^{\ b\ -}\Theta(-w) + T_a^{\ b\ +}\Theta(w) + \delta(w)S_a^{\ b}, \tag{9}$$

with $S_a^{\ b}$ being the energy-momentum tensor on the brane, and $T_a^{\ b\ \pm}$ being the energy-momentum tensors on the both sides of the brane.

Assume Gaussian normal coordinates, i.e., $(\mu, \nu = 0, 1, 2, \ldots, D-2; w = D)$

$$ds^2 = g_{ab}dx^a dx^b = \epsilon dw^2 + h_{\mu\nu}dx^\mu dx^\nu \ , \tag{10}$$

where $\epsilon = \vec{n} \cdot \vec{n} = +1$ for a spacelike hypersurface, $\epsilon = -1$ for a timelike hypersurface, and $h_{ab} = g_{ab} - \epsilon n_a n_b$ is a projection tensor onto a $(D-1)$-dimensional hypersurface, $\vec{n}$ is the normal vector to the hypersurface. In these coordinates the extrinsic curvature is

$$K_{\mu\nu} = -\frac{1}{2}\frac{\partial h_{\mu\nu}}{\partial w} \ , \tag{11}$$

and the Gauss-Codazzi equations read

$$R_{w\mu w\nu} = \frac{\partial K_{\mu\nu}}{\partial w} + K_{\rho\nu}K^{\rho}{}_{\mu}, \quad (12)$$

$$R_{w\mu\nu\rho} = \nabla_{\nu}K_{\mu\rho} - \nabla_{\rho}K_{\mu\nu}, \quad (13)$$

$$R_{\lambda\mu\nu\rho} = {}^{(D-1)}R_{\lambda\mu\nu\rho} + \epsilon\left[K_{\mu\nu}K_{\lambda\rho} - K_{\mu\rho}K_{\lambda\nu}\right] . \quad (14)$$

In the standard Israel approach one assumes that at the brane position $w = 0$:

$$h^{-}_{\mu\nu} = h^{+}_{\mu\nu} , \quad (15)$$

$$h^{-}_{\mu\nu,w} \neq h^{+}_{\mu\nu,w} , \qquad K^{-}_{\mu\nu} \neq K^{+}_{\mu\nu} , \quad (16)$$

i.e., the metric is continuous but it has a kink, its first derivative has a step function discontinuity, and its second derivative gives the delta function contribution.

In terms of the appropriate quantities this is equivalent to

$$h_{\mu\nu}(w) = h^{-}_{\mu\nu}(w)\theta(-w) + h^{+}_{\mu\nu}(w)\theta(w) \; , \tag{17}$$

$$\frac{\partial h_{\mu\nu}}{\partial w} = \frac{\partial h^{+}_{\mu\nu}}{\partial w}\theta(-w) + \frac{\partial h^{-}_{\mu\nu}}{\partial w}\theta(w) \; , \tag{18}$$

$$\begin{aligned} \frac{\partial^2 h_{\mu\nu}}{\partial w^2} &= \frac{\partial^2 h^{-}_{\mu\nu}}{\partial w^2}\theta(-w) + \frac{\partial^2 h^{+}_{\mu\nu}}{\partial w^2}\theta(w) \\ &+ \left(\frac{\partial h^{-}_{\mu\nu}}{\partial w} - \frac{\partial h^{+}_{\mu\nu}}{\partial w}\right)\delta(w) \; . \end{aligned} \tag{19}$$

For the standard brane models with the Einstein-Hilbert action in the bulk

$$S = \frac{1}{2\kappa^2}\int_M d^D x \sqrt{-g} R + S_{brane} + S_m \tag{20}$$

the field equations read as

$$G^w_{\ w} = -\frac{1}{2}\,{}^{(D-1)}R + \frac{1}{2}\epsilon\left[K^2 - Tr(K^2)\right] = \kappa^2 T^w_{\ w}, \tag{21}$$

$$G^w_{\ \mu} = \epsilon\left[\nabla_\mu K - \nabla_\nu K^\nu_{\ \mu}\right] = \kappa^2 T^w_{\ \mu}, \tag{22}$$

$$\begin{aligned} G^\mu_{\ \nu} &= {}^{(D-1)}G^\mu_{\ \nu} + \epsilon\left[\frac{\partial K^\mu_{\ \nu}}{\partial w} - \delta^\mu_{\ \nu}\frac{\partial K}{\partial w}\right] \\ &+ \epsilon\left[-KK^\mu_{\ \nu} + \frac{1}{2}\delta^\mu_{\ \nu}Tr(K^2) + \frac{1}{2}\delta^\mu_{\ \nu}K^2\right] = \kappa^2 T^\mu_{\ \nu}\,. \end{aligned} \tag{23}$$

and in the limit $\lim_{w\to 0}\int_{-w}^{w}$ which fishes out the delta function contributions one gets the standard Israel junction conditions as:

$$\epsilon\{[K^\mu_{\ \nu}] - \delta^\mu_{\ \nu}[K]\} = \kappa^2 S^\mu_{\ \nu}, \qquad [K^\mu_{\ \nu}] \equiv K^\mu_{\ \nu}{}^+ - K^\mu_{\ \nu}{}^-. \tag{24}$$

However, for the general $f(X,Y,Z)$ theory on the brane the standard continuity relations do not work. This can be seen from the field equations of the action (8)

$$
\begin{aligned}
P_{ab} &= \frac{\chi}{2} T_{ab}, && (25)\\
P^{ab} &= -\frac{1}{2} f g^{ab} + f_X R^{ab} + 2 f_Y R^{c(a} R^{b)}{}_c + 2 f_Z R^{edc(a} R^{b)}{}_{cde} \\
&+ f_{X;cd}(g^{ab}g^{cd} - g^{ac}g^{bd}) + \Box(f_Y R^{ab}) + g^{ab}(f_Y R^{cd})_{;cd} \\
&- 2(f_Y R^{c(a})_{;}{}^{b)}{}_c - 4(f_Z R^{d(ab)c})_{;cd}, && (26)
\end{aligned}
$$

where $f_X = \partial f/\partial X$ etc.

Take for example the square of the Ricci scalar

$$
R = {}^{(D-1)}R + \epsilon \left[2h^{\mu\nu} \frac{\partial K_{\mu\nu}}{\partial w} + 3Tr(K^2) - K^2 \right],
$$

where $K \equiv K^{\mu}_{\ \mu}$, $Tr(K^2) \equiv K^{\mu\nu}K_{\mu\nu}$, (and appropriately, of the Ricci tensor, and of the Riemann tensor). In fact, the terms of the type

$$\frac{\partial^2 h^{\mu\nu}}{\partial^2 w}\frac{\partial K_{\mu\nu}}{\partial w}, \frac{\partial K_{\mu\nu}}{\partial w}\frac{\partial K^{\mu\nu}}{\partial w}, \left(\frac{\partial K}{\partial w}\right)^2 \ , \qquad (27)$$

are proportional to $\delta^2(w)$ and are ambiguous.

Amazingly, all these ambiguous terms exactly cancel in the case of Euler densities (Meissner & Olechowski '01). In fact, the junction conditions for the Gauss-Bonnet density were already obtained as (Deruelle and Doležel 2000, Davis 2003)

$$2\alpha\left(3[J_{\mu\nu}] - [J]h_{\mu\nu} - 2[P]_{\mu\rho\nu\sigma}[K]^{\rho\sigma}\right) + [K_{\mu\nu}] - [K]h_{\mu\nu} = -\kappa^2 S_{\mu\nu} \ , \qquad (28)$$

where

$$P_{\mu\rho\nu\sigma} = R_{\mu\rho\nu\sigma} + 2h_{\mu[\sigma}R_{\nu]\rho} + 2h_{\rho[\nu}R_{\sigma]\mu} + Rh_{\mu[\nu}h_{\sigma]\rho} \ , \tag{29}$$

$$J_{\mu\nu} = \frac{1}{3}\left(2KK_{\mu\sigma}K^{\sigma}_{\nu} + K_{\sigma\rho}K^{\sigma\rho}K_{\mu\nu} - 2K_{\mu\rho}K^{\rho\sigma}K_{\sigma\nu} - K^2K_{\mu\nu}\right) \tag{30}$$

In view of that we found two ways in formulating the junction conditions for general $f(X,Y,Z)$ theories on the brane:

- A. Smoothing the continuity conditions for the metric tensor at the brane
- B. Considering an equivalent 2nd order theory (transition to the Einstein frame)

A. Smoothing out the continuity conditions

We impose more regularity onto the metric tensor at the brane position - consider **singular hypersurface of order three** (Israel 1966)

$$
\begin{aligned}
h^{-}_{\mu\nu} &= h^{+}_{\mu\nu} \, , && (31)\\
h^{-}_{\mu\nu,w} &= h^{+}_{\mu\nu,w} \, , \qquad K^{-}_{\mu\nu} = K^{+}_{\mu\nu} \, , && (32)\\
h^{-}_{\mu\nu,ww} &= h^{-}_{\mu\nu,ww} \, , \qquad K^{-}_{\mu\nu,w} = K^{+}_{\mu\nu,w} \, , && (33)\\
h^{-}_{\mu\nu,www} &\neq h^{+}_{\mu\nu,www} \, , \qquad K^{-}_{\mu\nu,ww} \neq K^{+}_{\mu\nu,ww} \, , && (34)
\end{aligned}
$$

i.e., the metric and its first derivative are regular, the second derivative of the metric is continuous, but possesses a kink, the third derivative of the metric has a step function discontinuity, and no sooner than the fourth derivative of the metric on the brane produces the delta function contribution.

The physical interpretation is that there can be a jump of the first derivative of the energy-momentum tensor (e.g. jump of a pressure gradient).

smoothing out the continuity conditions

In his seminal work, Israel (1966) proposed **a singular hypersurface of order two** which physically corresponded to a boundary surface characterized by a jump of the energy-momentum tensor (e.g. a boundary surface separating a star from the surrounding vacuum) which was characterized by

$$h^{-}_{\mu\nu} = h^{+}_{\mu\nu} , \tag{35}$$

$$h^{-}_{\mu\nu,w} = h^{+}_{\mu\nu,w} , \qquad K^{-}_{\mu\nu} = K^{+}_{\mu\nu} , \tag{36}$$

$$h^{-}_{\mu\nu,ww} \neq h^{-}_{\mu\nu,ww} , \qquad K^{-}_{\mu\nu,w} = K^{+}_{\mu\nu,w} , \tag{37}$$

i.e., the metric is regular, the first derivative of the metric is continuous, but possesses a kink, and the second derivative of the metric has a step function discontinuity, and the third derivative of the metric on the brane produces the delta function contribution.

The appropriate junction conditions can be obtained as follows:

We rewrite the field equations (25)-(26) as

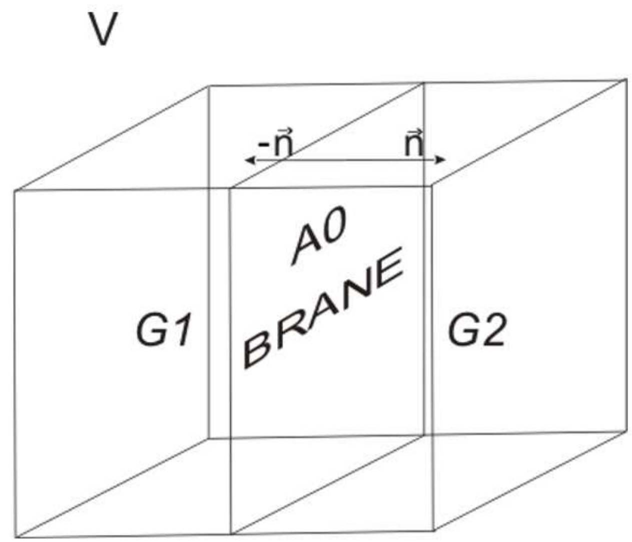
$$\sqrt{-g}C_{ab}W^{abd}{}_{;d} + \sqrt{-g}C_{ab}V^{ab} = \frac{\chi}{2}T^{ab}C_{ab}\sqrt{-g}\ , \tag{38}$$

where we have introduced is an arbitrary tensor field C_{ab}, and

$$\begin{aligned} W^{abd} &= f_{X;c}(g^{ab}g^{cd} - g^{(ac}g^{b)d}) + (f_Y R^{ab})^{;d} \\ &+ g^{ab}(f_Y R^{cd})_{;c} - 2(f_Y R^{d(a})^{;b)} - 4(f_Z R^{d(ab)c})_{;c}\ , \\ V^{ab} &= -\frac{1}{2}fg^{ab} + f_X R^{ab} + 2f_Y R^{c(a}R^{b)}{}_c \\ &+ 2f_Z R^{edc(a}R^{b)}{}_{cde}\ , \end{aligned} \tag{39, 40}$$

contain third derivatives of the metric giving a step function discontinuity and so $W^{abd}{}_{;d}$ is proportional to $\delta(w)$. We integrate both sides of the formula (38) over the volume V.

We have: $V = G1 + G2$, $A1 = \partial G1 + A0$ and $A2 = \partial G2 - A0$ (see below).



$$\int_{G1+G2} \sqrt{-g} C_{ab} W^{abd}{}_{;d} d\Omega \qquad (41)$$

$$+ \int_{G1+G2} \sqrt{-g} C_{ab} V^{ab} d\Omega = \int_{G1+G2} \frac{\chi}{2} T^{ab} C_{ab} \sqrt{-g} d\Omega \ ,$$

and so

$$
\begin{aligned}
& \int_{G1+G2} \sqrt{-g}(C_{ab}W^{abd})_{;d} d\Omega \\
- & \int_{G1+G2} \sqrt{-g} C_{ab;d} W^{abd} d\Omega \\
+ & \int_{G1+G2} \sqrt{-g} C_{ab} V^{ab} d\Omega \\
= & \int_{G1} \frac{\chi}{2} T^{ab} C_{ab} \sqrt{-g} d\Omega + \int_{G2} \frac{\chi}{2} T^{ab} C_{ab} \sqrt{-g} d\Omega \\
+ & \int_{A0} \frac{\chi}{2} S^{ab} C_{ab} \sqrt{-\gamma} d\sigma \ ,
\end{aligned}
\tag{42}
$$

of which first term can be integrated out to a boundary A1+A2 and then the limit $V \to A0$ (or $\lim_{w\to 0} \int_{-w}^{w}$ in Gaussian coordinates) is taken.

The appropriate junction conditions are obtained as:

$$[W]^{abd}n_d - \frac{\chi}{2}S^{ab} \quad = \quad 0\,, \quad [W]^{abd} = W^{abd+} - W^{abd-}. \tag{43}$$

It is remarkable that these junction conditions involve the higher derivatives of the scale factor. To see this take for example $f(X,Y,Z) = f(R)$ theory in $D = 5$ dimensions with metric

$$ds^2 = -dt^2 + a^2(t,w)[dr^2 + r^2(d\Theta^2 + sin^2\Theta d\phi^2)] + dw^2 \ .$$

The junction conditions (43) give a jump in the third derivative as expected

$$[a'''] \quad = \quad \frac{\chi}{2}a_0 p_0\ , \tag{44}$$

$$p_0 \quad = \quad \rho_0\ , \tag{45}$$

where $(\ldots)' = \partial/\partial w$, $a_0 = a(w=0)$ and the brane energy-momentum tensor $S^\nu_\mu = (-\rho_0, p_0, p_0, p_0)$.

B. Reduction to an equivalent 2nd order theory

gives equivalent junction conditions though at the expence of introducing a new tensor field H^{abcd} (tensoron). Starting from (Kijowski & Jakubiec '88)

$$S_G = \chi^{-1} \int_M d^D x \sqrt{-g} f(g_{ab}, R_{abcd}). \tag{46}$$

we may consider an equivalent 2nd order action

$$S_I = \chi^{-1} \int_M d^D x \sqrt{-g} \{ H^{ghij} (R_{ghij} - \phi_{ghij}) + f(g_{ab}, \phi_{cdef}) \} \,, \tag{47}$$

where

$$H^{ghij} \equiv \frac{\partial f(g_{ab}, \phi_{abcd})}{\partial \phi_{ghij}} \,, \qquad det \left[\frac{\partial^2 f(g_{ab}, \phi_{abcd})}{\partial \phi_{ghij} \partial \phi_{klmn}} \right] \neq 0. \tag{48}$$

(For $f(R)$ theory one defines the scalar $H = f'(Q)$ **(a scalaron)**; similarly for $f(R_{GB})$ one defines $H = f'(A)$ with the equation of motion $A = R_{GB}$).

We have to slightly redefine the tensoron

$$A^{abcd} = \frac{1}{2}\{H^{acdb} + H^{abdc} - H^{cbda} - H^{acbd} - H^{abcd} + H^{cbad}\} \quad (49)$$

which in a particular case of $f(X, Y, Z)$ theory takes the form

$$\begin{aligned} A^{abcd} &= f_X(g^{ad}g^{cb} - g^{cd}g^{ba}) \\ &+ f_Y(2R^{ad}g^{bc} - R^{cd}g^{ba} - R^{ba}g^{cd}) \\ &+ 4f_Z R^{acbd} \; . \end{aligned} \quad (50)$$

The field equations for an equivalent action read

$$R_{ghij} = -\frac{\partial V(g_{ab}, H^{cdef})}{\partial H^{ghij}} \,, \tag{51}$$

$$\begin{aligned} \frac{1}{2} g^{ab} f &+ \frac{\partial f}{\partial g_{ab}} + H^{becd} \phi^{a}{}_{ecd}(g_{ab}, H^{klmn}) + \\ &+ \{A^{(ab)cd}\}_{;dc} = -\frac{\chi}{2} T^{ab} \,, \end{aligned} \tag{52}$$

where

$$\begin{aligned} & V(g_{ab}, H^{cdef}) = \\ - \; & H^{hgij} \phi_{ghij}(g_{ab}, H^{cdef}) + f(g_{ab}, \phi_{klmn}(g_{ab}, H^{cdef})) \end{aligned} \tag{53}$$

In fact, the possibility to express the fields ϕ_{abcd} as a function of g_{ab} and H^{cdef} is guaranteed by the condition (48) (an analogue of the condition $f''(Q) \neq 0$).

j.c. of the second-order theory are equivalent to j.c. of the fourth-order.

Applying the same method as in the previous case (i.e. taking the limit of $V \to A0$) we notice that the first three terms of (52) do not give any contribution to the junction conditions (since they do not contain delta functions at all) which now have the form:

$$[A^{(ab)cd}{}_{;d}]n_c = -\frac{\chi}{2} S^{ab} \ . \tag{54}$$

Assuming that

$$f(g_{ab}, \phi_{abcd}) = f(\phi_{ab}{}^{ab}, \phi_{acb}{}^{c}\phi^{acb}{}_{c}, \phi_{abcd}\phi^{abcd}), \tag{55}$$

we can get the same result as in fourth-order theory

$$\begin{aligned} [A^{(ab)cd}{}_{;d}]n_c &= [A^{(ab)cd}{}_{;c}]n_d = [-\{f_{X;c}(g^{ab}g^{cd} - g^{c(a}g^{b)d}) \\ &+ (f_Y R^{ab})^{;d} + g^{ab}(f_Y R^{cd})_{;c} \\ -2(f_Y R^{d(a})^{;b)} &- 4(f_Z R^{d(ab)c})_{;c}\}]n_d = -[W^{abd}]n_d. \end{aligned} \tag{56}$$

Similar approach was used for less-general theories of gravity on the brane:

- $f(R)$ - Borzeszkowski and Frolov 1980; Parry, Pichler, Deeg JCAP 2005; Deruelle et al. gr-qc/0711.1150
- $f(X, Y, Z) = aX^2 + bY + cZ$ - Nojiri, Odintsov JHEP 2000; PRD 2001

3. Formulation of 4th order gravities on the brane - Gibbons-Hawking (GH) boundary terms.

- In this approach one does not assume vanishing of the first derivative of the variation of the metric tensor $\delta g_{ab;c}$ on the boundary of the integration volume. Instead one postulates some extra terms to be added to the action - these terms "kill" the first derivatives of the metric variation.
- For EH action a GH term is just composed of the extrinsic curvature.
- For the action being the combination of the square of the Weyl tensor and an arbitrary function of the scalar curvature they were found by Hawking and Lutrell (1984) and Barrow and Madsen (1989).
- For the Gauss-Bonnet and other Lovelock densities they were found by Bunch (1981), Mueller-Hoissen (1985), Myers (1987), Davis (2003) and Gravanis and Willinson (2006, 2007).
- For an arbitrary function of the curvature invariants were found by Barvinsky and Solodukhin (1996).

GH terms for $f(R)$ and $f(X,Y,Z)$ theories:

$$S_{GH,p} = -2(-1)^p \epsilon \int_{\partial M_p} \sqrt{-h} H K d^{D-1}x \ , \tag{57}$$

where $H = f'(Q)$ is the scalaron.

$$\begin{aligned} S_{GH,p} &= \\ &- (-1)^p \int_{\partial M_p} d^{D-1}x \sqrt{-h} A^{(ab)cd} n_c n_d \mathcal{L}_{\vec{n}} g_{ab} \ , \end{aligned} \tag{58}$$

where $A^{(ab)cd}$ is the tensoron.

The most general j.c. for $f(R)$ theory:

(no continuity of the scalaron on the brane was assumed)

$$
\begin{aligned}
[K] &= 0\,, && (59)\\
S^{ab} n_a n_b &= 0\,, && (60)\\
S^{ab} h_{ac} n_b &= 0\,, && (61)\\
-(D-1)[H_{;c}n^c] - D[H]K &= \epsilon \frac{\chi}{2} S^{ab} h_{ab}\,, && (62)\\
-h_{ab}[H_{;c}n^c] - [H]K h_{ab} &+ [HK_{ab}] && (63)\\
&= \epsilon \frac{\chi}{2} S^{cd} h_{ca} h_{db}.
\end{aligned}
$$

The most general j.c. for $f(X,Y,Z)$ theory:

(no assumption about the continuity of the tensoron on the brane)

$$
\begin{aligned}
& [KA^{(ab)cd}]n_c n_d + [\mathcal{L}_{\vec{n}} A^{(ab)cd}]n_c n_d \\
- \quad & \epsilon[A^{(ab)cd}K_{cd}] - g^{ab}[A^{(ef)cd}K_{ef}]n_c n_d \\
+ \quad & 2\epsilon[D_s A^{(ef)cd} n_c n_d] h^s_e h^{(a}_f n^{b)} - 2\epsilon[A^{(ab)cd}{}_{;(c}]n_{d)} = \frac{\chi}{2} S^{ab} , \\
& n_b n_c [\mathcal{L}_{\vec{n}} g_{ad}] - n_a n_c [\mathcal{L}_{\vec{n}} g_{db}] - n_b n_d [\mathcal{L}_{\vec{n}} g_{ac}] + n_a n_d [\mathcal{L}_{\vec{n}} g_{cb}] = 0 .
\end{aligned} \tag{64}
$$

They reduce to the conditions (54) if one assumes continuity of the tensoron on the brane

$$
[A^{(ab)cd}] = 0 . \tag{65}
$$

4. Fourth-order gravities and statefinders (jerk, snap etc.)

Statefinders are the higher-order characteristics of the universe expansion which go beyond the Hubble parameter and the deceleration parameter

$$H = \frac{\dot{a}}{a}, \qquad q = -\frac{1}{H^2}\frac{\ddot{a}}{a} = -\frac{\ddot{a}a}{\dot{a}^2} . \tag{66}$$

They can generally be expressed as ($i \geq 2$)

$$x^{(i)} = (-1)^{i+1}\frac{1}{H^i}\frac{a^{(i)}}{a} = (-1)^{i+1}\frac{a^{(i)}a^{i-1}}{\dot{a}^i} , \tag{67}$$

and the lowest order of them are known as: jerk, snap ("kerk"), crack ("lerk")

$$j = \frac{1}{H^3}\frac{\dddot{a}}{a} = \frac{\dddot{a}a^2}{\dot{a}^3}, \qquad k = -\frac{1}{H^4}\frac{\ddddot{a}}{a} = -\frac{\ddddot{a}a^3}{\dot{a}^4}, l = \frac{1}{H^5}\frac{a^{(5)}}{a} = \frac{a^{(5)}a^4}{\dot{a}^5}, \tag{68}$$

the and pop ("merk"), "nerk", "oerk", "perk" etc. (Harrison ’76, Landsberg ’76, Chiba ’98, Alam et al. ’03, Sahni et al. ’03, Visser ’04, Caldwell, Kamionkowski ’04, MPD+Stachowiak ’06, Dunajski and Gibbons ’08)

In the case of the 4th order gravities statefinders may become powerful tools to constrain such theories observationally since they enter observational relations in the lower orders of z (Poplawski '06, '07; Cappoziello et al. 0802.1583).

Apparently, a blow-up of statefinders may easily be linked to an emergence of exotic singularities in the universe.

5. Conclusion

- The formulation of the fourth-order gravity theories on the brane is nontrivial because of the powers of delta function ambiguities.
- Two methods were applied: **A.** Smoothing out the continuity conditions for the metric tensor at the brane; **B.** Considering an equivalent 2nd order theory. In both cases the Israel junction conditions have been obtained.
- The method of the GH boundary terms was also applied and the most general junction conditions (with no continuity of the scalaron and tensoron on the brane assumed) were obtained that way, too.
- Higher-order brane gravities contain higher-order derivatives of the geometric quantities (in a Friedmann model - the scale factor) which may manifest themselves in the higher-order characteristics of expansion such as statefinders (jerk, snap, lerk/crack, merk/pop).