Varying constant cosmologies and cosmic singularities

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Multicosmofun'12, 10 Sept 2012

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1. Introduction.

Pretty long story of varying constants theories:

Hermann Weyl (1919): electron radius/its gravitational radius $\sim 10^{40}$

Arthur Eddington (1935) discussed:

1) proton-to-electron mass $1/\beta = m_p/m_e \sim 1840$

2) an inverse of fine structure constant $1/\alpha = (hc)/(2\pi e^2) \sim 137$

3) electromagnetic to gravitational force between a proton and an electron $e^2/(4\pi\epsilon_0 G m_e m_p) \sim 10^{40}$

4) introduced "Eddington number" $N_{edd} \sim 10^{80}$

P.A.M. Dirac (1937) interesting remarks about the relations between atomic and cosmological quantities: If $G \propto H(t) = (da/dt)/a$, then $a(t) \propto t^{1/3}$ and $G(t) \propto 1/t$ - fundamental constants must evolve in time.

What could be benefits of these assumptions onto our view of cosmological singularities?

2. Varying constant cosmologies and their advantages.

First fully quantitative framework: Brans-Dicke scalar-tensor gravity (1961)

The gravitational constant G is associated with an average gravitational potential (scalar field) ϕ surrounding a given particle:

$< \phi >= GM/(c/H_0) \propto 1/G = 1.35 \times 10^{28} g/cm$. The scalar field gives the strength of gravity

$$G = \frac{1}{16\pi\Phi} \tag{1}$$

With the action

$$S = \int d^4x\sqrt{-g}\left(\Phi R - \frac{\omega}{\Phi}\partial_\mu\Phi\partial^\mu\Phi + \Lambda + L_m\right) \tag{2}$$

it relates to low-energy-effective superstring theory for $\omega = -1$

String coupling constant (running) $g_s = \exp(\phi/2)$ changes in time with ϕ - the dilaton and $\Phi = \exp(-\phi)$.

contd. varying constant cosmologies and their advantages.

Varying speed of light theories (VSL) (Albrecht, Magueijo 1998; Barrow 1998; Magueijo 2003):

$$c^4 = \psi(x^\mu) \tag{3}$$

and so the action is

$$S = \int d^4x \sqrt{-g} \left[\frac{\psi(R + 2\Lambda)}{16\pi G} + L_m + L_\psi \right] \tag{4}$$

Claimed to solve basic problems of standard cosmology: horizon problem, flatness problem and Λ-problem.

Varying fine structure constant α (or charge $e = e_0 \epsilon(x^\mu)$ theories (Webb et al. 1999, Sandvik 2002)

$$S = \int d^4x \sqrt{-g} \left(\psi R - \frac{\omega}{2} \partial_\mu \psi \partial^\mu \psi - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} e^{-2\psi} + L_m \right) \tag{5}$$

with $\psi = \ln \epsilon$ and $f_{\mu\nu} = \epsilon F_{\mu\nu}$.

3. Standard and exotic singularities in cosmology.

Basing only on standard Einstein-Friedmann equations which are two equations for three unknown functions of time $a(t), p(t), \varrho(t)$

$$\varrho = 3\left(\frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\right) , \qquad (6)$$

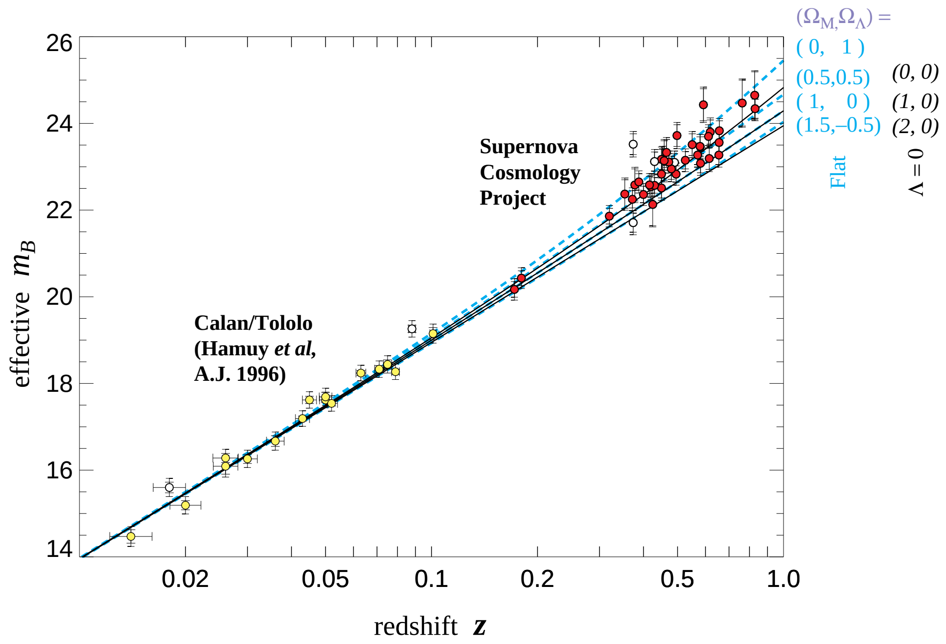
$$p = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\right) . \qquad (7)$$

with or without an equation of state, e.g., of a barotropic type ($w =$ const.):

$$p(t) = w\varrho(t) \qquad \Longrightarrow \qquad a(t) \propto t^{\frac{2}{3(w+1)}} . \qquad (8)$$

it is possible to get various types of singularities of different properties which are not standard Big-Bang or Big-Crunch.

Studies were motivated by supernovae observations ...

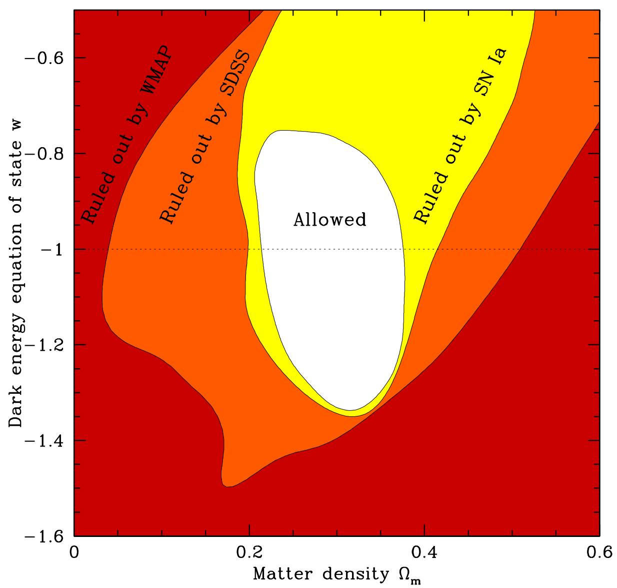


... which gave the evidence that not only the strong energy condition

$$\varrho + 3p \geq 0, \qquad \varrho + p \geq 0 \, . \tag{9}$$

Combined astronomical data.

... but also other energy conditions N(ull) E(nergy) C(ondition) $\varrho + p \geq 0$,
W(eak) E(nergy) C(ondition) $\varrho + p \geq 0, \rho \geq 0$,
D(ominant) E(nergy) C(ondition) $\mid p \mid \leq \varrho, \varrho \geq 0$
can be violated (phantom - Caldwell 1999, 2002)
Supported by WMAP + SDSS + Supernovae **combined bound on the dark energy barotropic index** w (Tegmark et al. (2004)):



Classification of exotic singularities (Nojiri, Odintsov, Tsujikawa 2005).

- Type 0 - Big-Bang $a \to 0$, $p \to \infty$, $\varrho \to \infty$
- Type I - Big-Rip $a \to \infty$, $p \to \infty$, $\varrho \to \infty$ (Caldwell 2002)
- Type II - Sudden Future (includes Big Boost and Big-Brake) $a =$ const., $\varrho =$ const., $p \to \infty$ (Barrow 2004)
- Type IIg - Generalized Sudden Future a= const., $\varrho =$ const., $p =$const., $\dddot{a} \to \infty$ etc., $w < \infty$ (Barrow 2004)
- Type III - Finite Scale Factor (also Big-Freeze) $a = a_s$ = const., $\varrho \to \infty$, $p \to \infty$ (NOT 2005)
- Type IV - Big Separation: a= const., $p = \varrho = 0$, $w \to \infty$, $\dddot{a} \to \infty$ etc. (NOT 2005) (and generalizations $p = \varrho =$const. Yurov 2010)
- Type V - w-singularity a= const., $p = \varrho = 0$, $w \to \infty$ (MPD, Denkiewicz 2009) (and generalizations $p =$const. Yurov 2010)
- Little-Rip, Pseudo-Rip (Frampton et al. 2011, 2012)

Are these really singularities - strength?

As an example let us take an SFS which is determined by a blow-up of the Riemann tensor and its derivatives.

Geodesics do not feel SFSs at all, since geodesic equations are not singular for $a_s = a(t_s) =$ const. (Fernandez-Jambrina, Lazkoz PRD 74, 064030 (2006))

$$\left(\frac{dt}{d\tau}\right)^2 = A + \frac{P^2 + KL^2}{a^2(t)} , \tag{10}$$

$$\frac{dr}{d\tau} = \frac{P_1 cos\phi + P_2 \sin\phi}{a^2(t)} \sqrt{1 - Kr^2} , \tag{11}$$

$$\frac{d\phi}{d\tau} = \frac{L}{a^2(t) r^2} . \tag{12}$$

Geodesic deviation equation

$$\frac{D^2 n^\alpha}{d\lambda^2} + R^\alpha_{\beta\gamma\delta} u^\beta n^\gamma u^\delta = 0 , \tag{13}$$

feels SFS since at $t = t_s$ we have the Riemann tensor $R^\alpha_{\beta\gamma\delta} \to \infty$.

Classification of exotic singularities - strength.

- Tipler's (Phys. Lett. A64, 8 (1977)) definition (of a weak singularity):
  $\int_0^{\tau} d\tau' \int_0^{\tau'} d\tau'' R_{ab} u^a u^b$
  does not diverge on the approach to a singularity at $\tau = \tau_s$
- Królak's (CQG 3, 267 (1988)) definition (of a weak singularity):
  $\int_0^{\tau} d\tau' R_{ab} u^a u^b$
  does not diverge on the approach to a singularity at $\tau = \tau_s$
- Type 0 (BB, BC): T, K - strong
- Type I (BR): T, K - strong
- Type II (SFS): T, K - weak
- Type IIg (GSFS): T, K - weak
- Type III (FSF): T - weak, K - strong
- Type IV (BS): T, K - weak
- Type V (w-sing.): T, K - weak (Fernandez-Jambrina (PRD, 2010)

SFS (type II) - scale factor.

One chooses the scale factor in the field equations only (Barrow 2004):

$$a(t) = a_s \left[\delta + (1-\delta)\, y^m - \delta\, (1-y)^n\right] \; , \qquad y \equiv \frac{t}{t_s} \tag{14}$$

The parameter m can be taken to be just a form of the w parameter present in the

barotropic equation of state: – $0 < m \leq 1$ when $w \geq -1/3$ (standard matter - Big-Bang);

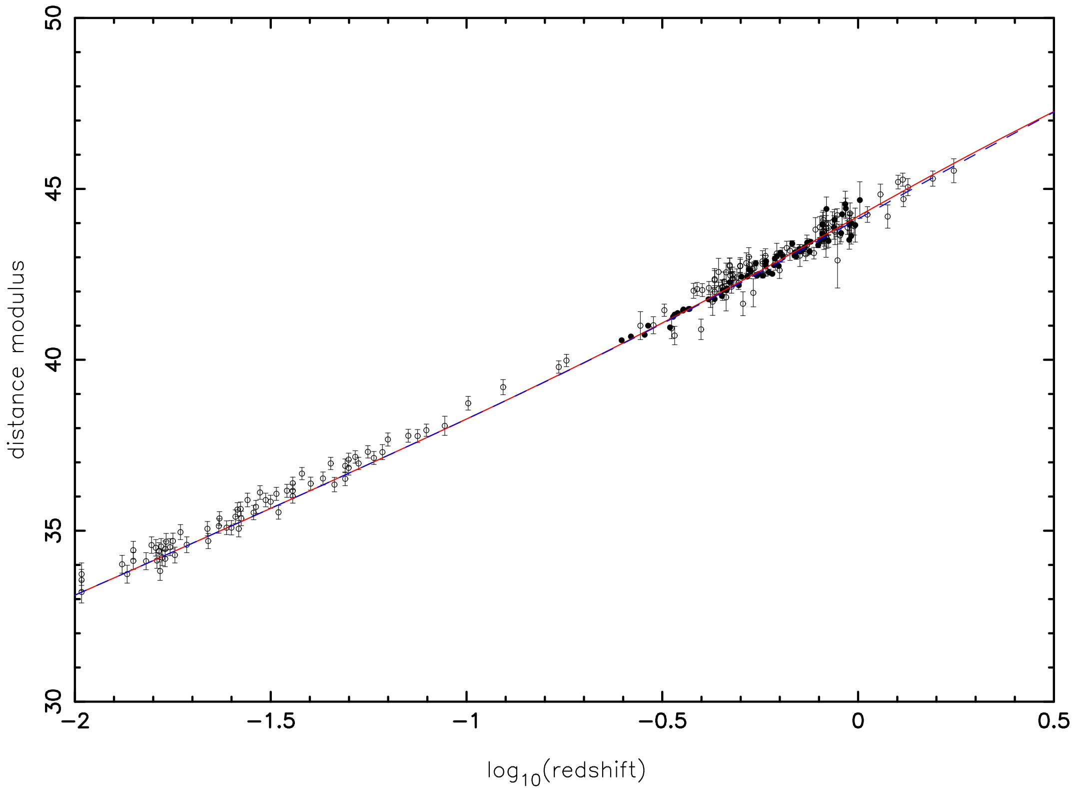
– $m > 1$ when $-1 < w < -1/3$ (quintessence - Big-Bang);

– $m < 0$ when $w < -1$ (phantom - Big-Rip). An SFS singularity is obtained for

$1 < n < 2$;

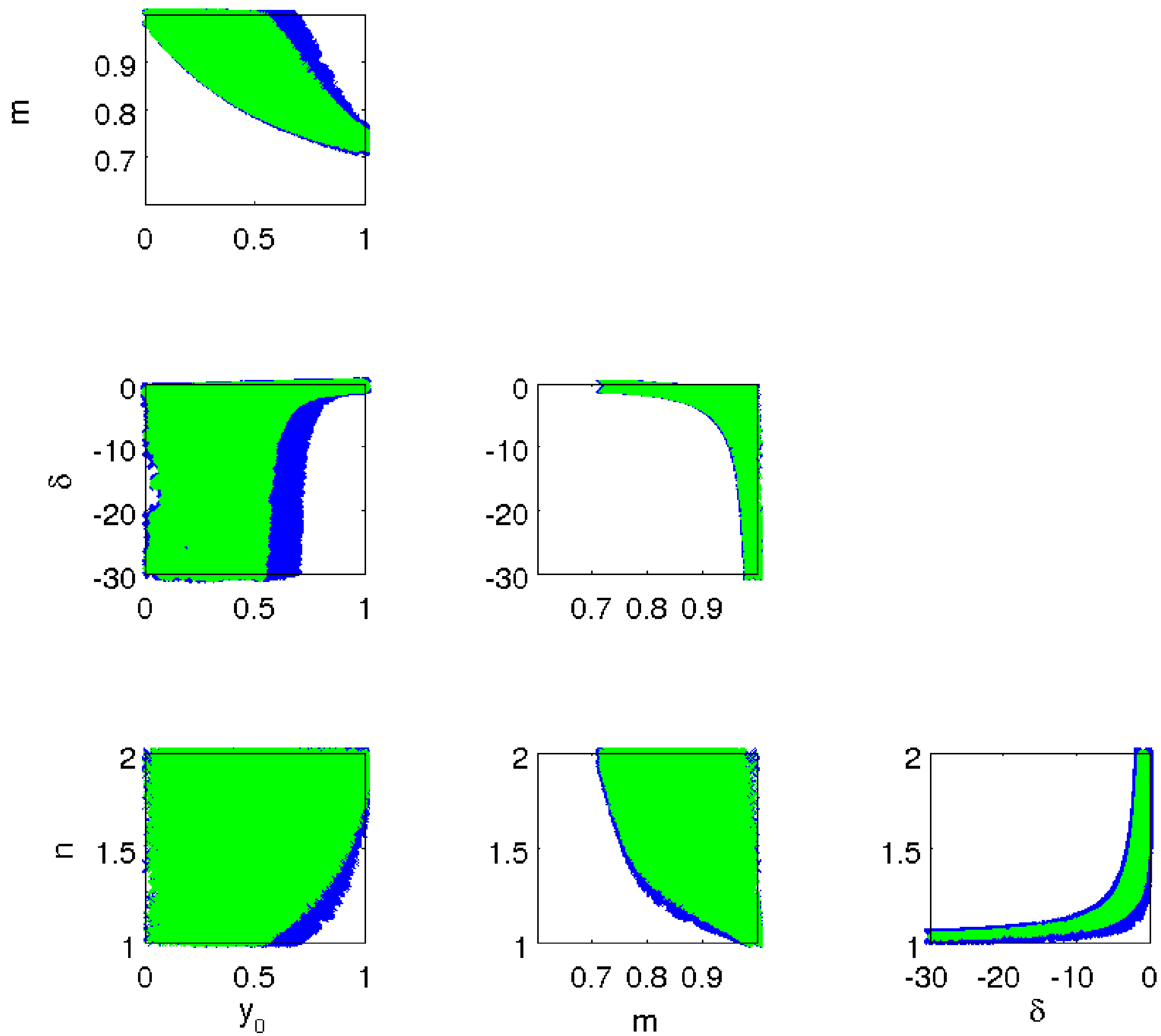
An FSF singularity is obtained for $0 < n < 1$.

SFS dark energy mimics Λ-term (supernovae only)



Distance modulus $\mu_L = m - M$ for the CC model ($H_0 = 72\text{kms}^{-1}\text{Mpc}^{-1}$, $\Omega_{m0} = 0.26$, $\Omega_{\Lambda 0} = 0.74$) (dashed curve) and SFS model ($m = 2/3 = 0.6666, n = 1.9999, \delta = -0.471, y_0 = 0.99936$) (solid curve). Open circles are for the 'Gold' data and filled circles are for SNLS data.

Combined bound: supernovae, CMB shift parameter and BAO - fits if $m \approx 0.72$, $w = -0.82$. (Denkiewicz et al. 2012)



4. The universe through a singularity - averaging surprises.

A.K. Raychaudhuri (PRL 80, 654 (1998)) proposed that one may average physical and kinematical scalars over the whole open spacetime provided they vanish rapidly at spatial and temporal infinity as follows

$$< \chi >= \lim_{x^a \to \infty} \frac{\int\int\int\int_{-x^a}^{x^a} \chi\sqrt{-g}d^4x}{\int\int\int\int_{-x^a}^{x^a} \sqrt{-g}d^4x} \qquad (15)$$

By an open model it is meant that the ratio of the 3-volume hypersurfaces to a 4-volume of spacetime vanishes, i.e.,

$$\frac{\int\int\int \sqrt{|^3 g\,|}d^3x}{\int\int\int\int \sqrt{-g}d^4x} = 0. \qquad (16)$$

His idea was to tight the vanishing of the average $< \chi >$ with the singularity avoidance in cosmology.

Spacetime averaging - density and pressure.

For the pressure, the energy density, and the average acceleration we have

$$< p >= - \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} \frac{\int_{t_0}^{t_1} a^3 \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt} \tag{17}$$

and

$$< \varrho >= \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} \frac{3 \int_{t_0}^{t_1} a^3 \left(\frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt}. \tag{18}$$

$$< \dot{\theta} >= \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} \frac{3 \int_{t_0}^{t_1} a^3 \left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt}. \tag{19}$$

Subtle differences between singularities.

- BB, BC singularities - all the energy conditions fulfilled, averages vanish (despite original claim of Raychaudhuri)
- BR singularity - no EC fulfilled, averages blow up
- SFS - only dominant energy violated, averages finite
- It seems that BR is stronger singularity that BB, BC on the ground of averaging.
- SFS is weaker, but FSF does not seem so.

This seems to be a new kind of a measure for the strength of singularities.

5. Varying constants versus cosmic singularities.

We consider the Friedmann universes in varying speed of light (VSL) theories and varying gravitational constant G theories as follows

$$\varrho(t) = \frac{3}{8\pi G(t)}\left(\frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2}\right) , \tag{20}$$

$$p(t) = -\frac{c^2(t)}{8\pi G(t)}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2}\right) , \tag{21}$$

and the energy-momentum conservation law is

$$\dot{\varrho}(t) + 3\frac{\dot{a}}{a}\left(\varrho(t) + \frac{p(t)}{c^2(t)}\right) = -\varrho(t)\frac{\dot{G}(t)}{G(t)} + 3\frac{kc(t)\dot{c}(t)}{4\pi G a^2} . \tag{22}$$

New form of the scale factor.

We propose a new form of the scale factor, which admits big-bang, big-rip, sudden future, finite scale factor and w-singularities and reads as

$$a(t) = a_s \left(\frac{t}{t_s} \right)^m \exp \left(1 - \frac{t}{t_s} \right)^n , \tag{23}$$

with the constants t_s, a_s, m, n. For $k = 0$ we have

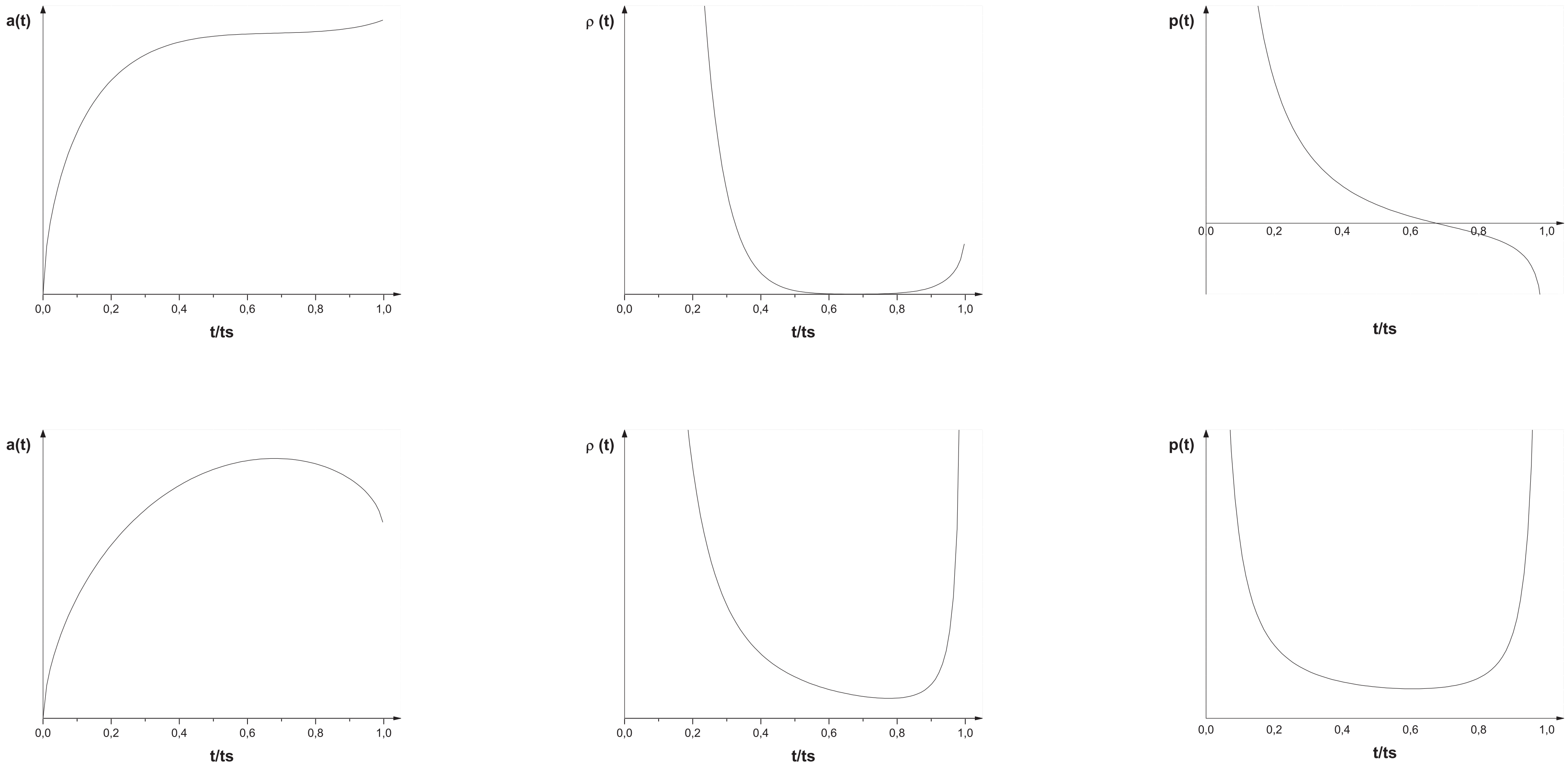
$$\varrho(t) = \frac{3}{8\pi G(t)} \left[\frac{m}{t} - \frac{n}{t_s} \left(1 - \frac{t}{t_s} \right)^{n-1} \right]^2 , \tag{24}$$

$$\begin{aligned} p(t) &= -\frac{c^2(t)}{8\pi G(t)} \left[\frac{m(3m-2)}{t^2} - 6\frac{mn}{tt_s} \left(1 - \frac{t}{t_s} \right)^{n-1} \right. \\ &+ \left. 3\frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s} \right)^{2(n-1)} + 2\frac{n(n-1)}{t_s^2} \left(1 - \frac{t}{t_s} \right)^{n-2} \right] . \end{aligned} \tag{25}$$

Contd. - new form of the scale factor.

For $0 < m < 2/3$ we have a big-bang singularity - $a \to 0$, $\varrho \to \infty$, $p \to \infty$ at $t \to 0$;
For $m < 0$ we have a big-rip singularity - $a \to \infty$, $\varrho \to \infty$, $p \to \infty$ at $t = 0$;
For $1 < n < 2$ we have a sudden future singularity (SFS) which appears at $t = t_s$ ($a = a_s$, $\varrho =$ const., $p \to \infty$);
For $0 < n < 1$ we have a stronger finite scale factor singularity (FSF) at $t = t_s$ ($a = a_s$, $\varrho \to \infty$, $p \to \infty$).
The plots of the scale factor $a(t)$, the energy density $\varrho(t)$, and the pressure $p(t)$ are given in Fig. (next page) for the two specific models. The upper plots are for the parameters $m = 0.6$, $n = 1.5$ and describe the sudden future singularity (SFS) while lower plots are for the parameters $m = 0.6$ and $n = 0.5$ and describe the finite scale factor singularity (FSF).
In fact, for $1 < n < 2$ only the last term in the pressure of the type $(1 - t/t_s)^{n-2}$ blows-up, while for $0 < n < 1$ two more terms $(1 - t/t_s)^{n-1}$ and $(1 - t/t_s)^{2(n-1)}$ do.

New form of the scale factor - plots



Regularizing singularities by varying constants

New idea: to change or even regularize various cosmological singularities by the variation of physical constants such as G, c, α etc.

One bears in mind the scale factor (23), the energy density (24) and pressure (25)

Regularizing a Big-Bang singularity by varying G:
If

$$G(t) \propto \frac{1}{t^2} \tag{26}$$

which is a faster decrease than in Dirac's LNH $G \propto 1/t$, then both divergence in ϱ and p are removed, though at the expense of having the "singularity" of gravitational coupling $G \to \infty$ at $t \to 0$.

contd. - regularizing singularities by varying constants: SFS

Regularizing an SFS singularity by varying c:

If

$$c(t) = c_0 \left(1 - \frac{t}{t_s}\right)^{\frac{p}{2}} , \tag{27}$$

then

$$\begin{aligned} p(t) &= -\frac{c_0^2}{8\pi G} \left[\frac{m(3m-2)}{t^2} \left(1 - \frac{t}{t_s}\right)^{p} \right. \\ &- 6\frac{mn}{tt_s} \left(1 - \frac{t}{t_s}\right)^{p+n-1} + 3\frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{p+2n-2} \\ &+ \left. 2\frac{n(n-1)}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{p+n-2} \right] . \end{aligned} \tag{28}$$

and the singularity of pressure is regularized provided

$$p > 2 - n \qquad (1 < n < 2) . \tag{29}$$

contd. - regularizing singularities by varying constants: w-sing.

In the limit $m \to 0$ we have an exotic singularity scale factor given by $a(t) = a_s \exp\left(1 - t/t_s\right)$ and so from (24) and (25) we have

$$\varrho_{ex}(t) = \frac{3}{8\pi G(t)} \frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{2(n-1)} , \tag{30}$$

$$p_{ex}(t) = -\frac{c^2(t)}{8\pi G(t)} \left[3\frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{2(n-1)} + 2\frac{n(n-1)}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{n-2} \right] \tag{31}$$

so that

$$w_{ex}(t) = \frac{p_{ex}(t)}{\varrho_{ex}(t)} = -c^2(t) \left[1 + \frac{2}{3} \frac{n-1}{n} \frac{1}{\left(1 - \frac{t}{t_s}\right)^n} \right] \tag{32}$$

which is a w-singularity for $n > 2$ ($p = \varrho = 0$, $w_{ex} \to \infty$). Its regularization by (27) is possible for

$$p > n \ . \tag{33}$$

contd. - regularizing singularities by varying constants: SFS

Regularizing an SFS singularity by varying G:

If we assume that

$$G(t) = G_0 \left(1 - \frac{t}{t_s}\right)^{-r} , \qquad (34)$$

($r =$ const., $G_0 =$ const.) which changes (24) and (25) to

$$\begin{aligned} \varrho(t) &= \frac{3}{8\pi G_0} \left[\frac{m^2}{t^2} \left(1 - \frac{t}{t_s}\right)^r - \frac{2mn}{tt_s} \left(1 - \frac{t}{t_s}\right)^{r+n-1} \right. \\ &+ \left. \frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{r+2n-2} \right] , \qquad (35) \\ p(t) &= -\frac{c^2}{8\pi G_0} \left[\frac{m(3m-2)}{t^2} \left(1 - \frac{t}{t_s}\right)^r - 6\frac{mn}{tt_s} \left(1 - \frac{t}{t_s}\right)^{r+n-1} + 3\frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{r+2n-2} \right. \\ &+ \left. 2\frac{n(n-1)}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{r+n-2} \right] . \qquad (36) \end{aligned}$$

contd. - regularizing singularities by varying constants: SFS

From (35) and (36) it follows that an SFS singularity $(1 < n < 2)$ is regularized by varying gravitational constant when

$$r > 2 - n \ , \tag{37}$$

and an FSF singularity $(0 < 1 < n)$ is regularized when

$$r > 1 - n \ . \tag{38}$$

On the other hand, assuming that we have an SFS singularity and that

$$-1 < r < 0 \, , \tag{39}$$

we get that varying G may change an SFS singularity onto a stronger FSF singularity when

$$0 < r + n < 1 \, . \tag{40}$$

Regularizing singularities: (anti-)Chaplygin gas

The equation of state of the (anti-)Chaplygin gas reads as

$$p(t) = \pm \frac{A}{\varrho(t)} \qquad (A > 0) \ , \tag{41}$$

where the "-" sign refers to a Chaplygin gas while the "+" sign refers to an anti-Chaplygin gas case.

Inserting (41) into (22) gives

$$\dot{\varrho}(t) + 3\frac{\dot{a}}{a}\left(\frac{\varrho^2 c^2(t) \mp A}{\varrho(t)c^2(t)}\right) = -\varrho(t)\frac{\dot{G}(t)}{G(t)} + 3\frac{kc(t)\dot{c}(t)}{4\pi G(t)a^2} \ . \tag{42}$$

We assume both varying $G = G(t)$ and $c = c(t)$ though with zero curvature $(k = 0)$ as follows

$$\varrho(t)c(t) = B = \text{const.} \ , \tag{43}$$

contd. - regularizing singularities: (anti-)Chaplygin gas

The solution of (42) reads as

$$\varrho(t) a^{3\gamma}(t) G(t) = E = \text{const.}\ , \tag{44}$$

where we have defined

$$\gamma \equiv \frac{B^2 \mp A}{B^2} \tag{45}$$

Putting the standard big-bang scale factor $a(t) = (t/t_s)^{2/3\gamma}$, we finally have

$$\varrho(t) = \frac{E t_s^2}{t^2 G(t)}\ , \qquad p(t) = \mp \left(\frac{t}{t_s}\right)^2 G(t)\ , \tag{46}$$

which give $\varrho \to \infty$ and $p(0) = 0$ provided $G(0) = \text{const.} \neq 0$. The singularity at $t = 0$ in ϱ and p can be regularized by taking $G(t) \propto 1/t^2$ at the expense of having a non-zero constant pressure (cosmological term).

Physical subtleties:

- In order to regularize an SFS, FSF or a $w-$singularity by varying $c(t)$, the light should slow and eventually stop propagating at a singularity. Similar effects were found in dense solid state fluids.
- To regularize an SFS, FSF by varying gravitational constant $G(t)$ - the strength of gravity has to become infinite at a singularity. On the one hand, it is quite reasonable because of the requirement to overcome an infinite (anti-)tidal forces at the singularity, but on the other hand, it makes another singularity - a singularity of strong coupling for a physical field such as $G \propto 1/\Phi$. Such problems were already dealt with in superstring and brane cosmology where both the curvature singularity and a strong coupling singularity appeared (choice of coupling, quantum corrections).

5. Conclusions

- Currently one is able to differentiate quite a number of cosmological singularities with completely different properties - despite many of them are geodesically complete, they still lead to a blow-up of various physical quantities (scale factor, energy density, pressure, physical fields).
- Some of these singularities may serve as dark energy, especially if they are quite close in the near future. For example, an SFS may even appear in 8.7 Myr with no contradiction with bare supernovae data. It can be fitted to a combined SnIa, CMB and BAO data, but at the expense of admitting an approach to a Big-Bang by a fluid which is not exactly dust (m=0.66), but has a slightly negative pressure (m = 0.73 and so w = -0.09).
- An interesting proposal is to investigate how the singularities are influenced by varying physical constants. In particular, we may look for the answer if it is possible to "regularize" (remove infinities) or change these singularities and what are the physical consequences of such an action, because what we face is usually the new "singularity" in a physical constant/field which acts to remove/change the type of singularity.