Redshift drift in inhomogeneous pressure (Stephani) cosmology

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Content:

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1. Universe symmetries. Acceleration as back-reaction of inhomogeneities.

In the context of dark energy problem there has been more interest in the non-friedmannian models of the universe which could explain the acceleration only due to inhomogeneity. One of the strogest claims was that

we are living in a spherically symmetric void of density described by the Lemaître-Tolman-Bondi dust spheres model

J. Uzan, R. Clarkson, G.F.R. Ellis (PRL, **100**, 191303 (2008))

R.R. Caldwell and A. Stebbins (PRL, **100**, 191302 (2008))

C. Clarkson, B. Bassett and T. H-Ch. Lu (PRL, **101**, 011301 (2008))

and many others

fortunately commented by A. Krasiński, R.A. Sussmann, K. Bolejko, M.-N. Célérier (e.g recent ArXiv:1206.6026) that not only LTB is worth investigating.

In fact, there are **two ways** to get large-scale structure in cosmology:

perturb FRW models $\leftrightarrow$ **consider exact inhomogeneous models**

How symmetric is the universe?

- Einstein equations are **complicated** and to solve them we just **assume symmetries** (Occam's razor - if we play with simple symmetric models observationally, we do not need to bother about any more complicated ones).
- Why not to **paradigm** this by a fundamental principle - the **Copernican Principle** that we do not live in the center of the Universe (we really do not want to be special in the Universe).
- However, observations are **from one point** in the Universe and extend only onto the one (and unique) past light cone (hopefully not - redshift drift!).
- Even **CMB** we observe from one point - this **proves isotropy**, but not necessarily homogeneity (isotropy with respect to any point in the Universe).
- Should not we first **start with observations** of the cone and then make conclusions related to modeling the universe (cf. observational cosmology programme of G.F.R. Ellis and collaborators).

Center of the Universe?

- In other words - homogeneity **needs a check**.
- Suppose we have an inhomogeneous model of the Universe with the **same (small) number of parameters** as a homogeneous dark energy model and they both fit observations very well.
- Could we **differentiate** between these two models?
- The simplest inhomogeneous models are the models with only one or two parameters which are **spherically symmetric** (isotropic with respect to only one point).
- Some even argue (Clarkson and Barrett, CQG ’99, ’00) that even if we proved isotropy for all observers in the universe, it would still not be enough to prove homogeneity, unless we proved all the fluid components in the universe were comoving perfect fluids.

New paradigm of inhomogeneity - LTB void.

- In fact, even if we restrict ourselves to spherical symmetry then there are **two complementary models** of the universe and they can both mimic homogeneous dark energy models!
- **These are: the inhomogeneous density (dust shells) Lemaître-Tolman-Bondi (LTB) models and inhomogeneous pressure (gradient of pressure shells) Stephani models.**
- Apparently for some reasons **most of the researchers investigate the former** and only a few investigate the latter, though there is no special reason to do so.
- Besides, there are a lot of less symmetric or purely inhomogeneous models (Goode (1986), Szafron (1977), Szekeres (1975), Wainwright-Goode (1980), Ruiz-Senovilla (1992) etc.) to investigate as candidates for dark matter. See e.g. M.-N. Célérier (ArXiv:1206.6026).

New paradigm of inhomogeneity?

- I suggest investigating **at least a complement of LTB** - spherically symmetric Stephani model of pressure gradient which also possesses a generalization which is totally spacetime inhomogeneous.
- Actually, as a self-promotion I would say that me and M. Hendry (Ap.J. ’98) first compared an inhomogeneous model (no matter if density or pressure) of the Universe with observational data from supernovae and showed that they can be fitted.
- Despite inhomogeneous density (LTB) models were theoretically explored before (since Lemaître - 1933) only later they were tested observationally against supernovae (e.g. K. Tomita, Prog. Theor. Phys. 106, 929 (2001); K. Bolejko, astro-ph/0512103).
- I also suggest that **we should carry on investigating more general models** of the universe and, in particular, check if they can give exact matter distribution in the universe which could be **equivalent to standard FRW perturbed models.**

2. Complementary models of the spherically symmetric Universe

Let us consider **advantages of the inhomogeneous pressure models** and show that they may fit observations, so that they are a good candidate for explanation of cosmic acceleration by an inhomogeneity.

In order to make a **complementary analysis** with LTB models the following table proves useful:

	pressure	density
FRW	$p = p(t)$	$\varrho = \varrho(t)$
LTB	$p = p(t)$	$\varrho = \varrho(t, r)$ - nonuniform
Stephani	$p = p(t, r)$ - nonuniform	$\varrho = \varrho(t)$

SS Lemaître-Tolman-Bondi Universe

– is the only spherically symmetric solution of Einstein equations for **pressureless matter** ($T^{ab} = \varrho u^a u^b$) and no cosmological term (G. Lemaître, Ann. Soc. Sci. Brux. **A 53**, 51 (1933); R.C. Tolman, Proc. Natl. Acad. Sci., **20**, 169 (1934); H. Bondi MNRAS **107**, 410 (1947))

$$ds^2 = -dt^2 + \frac{R'^2}{1-K}dr^2 + R^2(d\theta^2 + sin^2\theta d\phi^2) \ , \tag{1}$$

where

$$R = R(t,r); \qquad R' = \partial R/\partial r; \qquad K = K(r) \ . \tag{2}$$

The Einstein equations reduce to

$$\dot{R}^2 = \frac{2M(r)}{R} - K(r); \qquad 2M' = \kappa \varrho R^2 R' \ , \tag{3}$$

and are solved by

SS Lemaître-Tolman-Bondi Universe

$$R(r,\eta) = \frac{M(r)}{K(r)}\Phi'(\eta); \qquad t(r,\eta) = T_0(r) + \frac{M(r)}{K^{3/2}(r)}\phi'(\eta) \ , \tag{4}$$

where for $K(r) < 0$ (hyperbolic), $K(r) = 0$ (parabolic), and $K(r) > 0$ (elliptic) appropriately ($K(r)$ is a spatially dependent "curvature index") we have

$$\Phi(\eta) = (\sinh\eta - \eta; \eta^3/6; \eta - \sin\eta) \ . \tag{5}$$

Regularity conditions:

- existence of a regular **center of symmetry** $r = 0$ – implies $R(t,0) = \dot{R}(t,0) = 0$ and $M(0) = M'(0) = K(0) = K'(0) = 0$ and $R' \to 1$.
- hypersurfaces of constant time are **orthogonal** to 4-velocity and are of topology S^3 – implies the existence of a second center of symmetry $r = r_c$ (with some ‘turning value’ $0 < r_{tv} < r_c$)
- a ‘shell-crossing’ singularity should be **avoided** – implies $R'(t,r) \neq 0$ except at turning values

SS Lemaître-Tolman-Bondi Universe

Kinematic characteristics of the model:

$$u_{a;b} = \frac{1}{3}\Theta h_{ab} + \sigma_{ab} \quad , \tag{6}$$

Expansion scalar:

$$\Theta = \frac{2\dot{R}}{R} + \frac{\dot{R}'}{R'} \, , \tag{7}$$

Shear tensor and scalar:

$$\sigma^{ab} = \Sigma\zeta^{ab}; \qquad \zeta^{ab} \equiv h^{ab} - 3v^a v^b; \tag{8}$$

$$\Sigma = \frac{1}{6}\sigma_{ab}\zeta^{ab} = -\frac{1}{3}\left(\frac{\dot{R}'}{R'} - \frac{\dot{R}}{R}\right) \quad , \tag{9}$$

and $v^a = \sqrt{h^{rr}}\delta^a_r$ is the unit vector orthogonal to u^a and to the 2-sphere orbits of $SO(3)$

In LTB models a Big-Bang is not necessarily instantenuous - different points start at different moments.

Friedmann limit is obtained for:

$$R(t,r) = a(t)r; \qquad M(r) = M_0 r^3; \qquad K(r) = k_0 r^2 \ , \tag{10}$$

SS Stephani Universe

– is the only spherically symmetric solution of Einstein equations for perfect-fluid energy-momentum tensor ($T^{ab} = (\varrho + p)u^a u^b + pg^{ab}$) which is **conformally flat** and **embeddable** in a 5-dimensional flat space (H. Stephani Commun. Math. Phys. **4**, 167 (1967); A. Krasiński, GRG **15**, 673 (1983)). After introducing a Friedmann-like time coordinate (cf. later) we have

$$ds^2 = -\frac{a^2}{\dot{a}^2}\frac{a^2}{V^2}\left[\left(\frac{V}{a}\right)^{\cdot}\right]^2 dt^2 + \frac{a^2}{V^2}\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] , \quad (11)$$

where

$$V(t,r) = 1 + \frac{1}{4}k(t)r^2 , \quad (12)$$

and $(\ldots)^{\cdot} \equiv \partial/\partial t$. The function $a(t)$ plays the role of a **generalized scale factor**, $k(t)$ has the meaning of **a time-dependent ”curvature index”**, and r is the radial coordinate.

SS Stephani Universe

The energy density and pressure are given by

$$\varrho(t) = 3\left[\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k(t)}{a^2(t)}\right] , \tag{13}$$

$$p(t,r) = \varrho(t)\left\{-1 + \frac{1}{3}\frac{\dot{\varrho}(t)}{\varrho(t)}\frac{\left[\frac{V(t,r)}{a(t)}\right]^{\cdot}}{\left[\frac{V(t,r)}{a(t)}\right]}\right\} \equiv w_{eff}(t,r)\varrho(t) , \tag{14}$$

and generalize the standard Einstein-Friedmann relations

$$\varrho = 3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) , \tag{15}$$

$$p = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) \tag{16}$$

to inhomogeneous models.

SS Stephani Universe

Kinematic characteristic of the model:

$$u_{a;b} = \frac{1}{3}\Theta h_{ab} - \dot{u}_a u_b \ , \qquad \dot{u} \equiv (\dot{u}_a \dot{u}^a)^{\frac{1}{2}} \ . \tag{17}$$

where u is the acceleration scalar and the acceleration vector

$$\dot{u}_r = \frac{\left\{ \frac{a^2}{\dot{a}^2} \frac{a^2}{V^2} \left[\left(\frac{V}{a} \right)^{\cdot} \right] \right\}_{,r}}{\frac{a^2}{\dot{a}^2} \frac{a^2}{V^2} \left[\left(\frac{V}{a} \right)^{\cdot} \right]} \tag{18}$$

while the expansion scalar is the same as in FRW model, i.e.,

$$\Theta = 3\frac{\dot{a}}{a} \ . \tag{19}$$

3. Inhomogeneous pressure models - properties.

The general Stephani metric reads as

$$ds^2 = - \frac{a^2}{\dot{a}^2}\frac{a^2}{V^2}\left[\left(\frac{V}{a}\right)^{\cdot}\right]^2 dt^2 + \frac{a^2}{V^2}\left[dx^2 + dy^2 + dz^2\right], \quad (20)$$

$$V(t,x,y,z) = 1 + \frac{1}{4}k(t)\left\{[x - x_0(t)]^2 + [y - y_0(t)]^2 + [z - z_0(t)]^2\right\},$$

and x_0, y_0, z_0 are arbitrary functions of time. This is just a generalization of the FRW metric in isotropic coordinates

$$ds^2 = -dt^2 + \frac{a^2(t)}{1 + \frac{1}{4}kr^2}\left(dr^2 + r^2 d\Omega^2\right); \qquad r^2 = x^2 + y^2 + z^2 \quad (21)$$

which by a transformation $\bar{r} = 1 + (1/2)kr^2$ can be brought to a standard form

$$d\bar{s}^2 = -dt^2 + a^2(t)\left(\frac{d\bar{r}^2}{1 - k\bar{r}^2} + \bar{r}^2 d\Omega^2\right). \quad (22)$$

Inhomogeneous pressure models - properties

Properties of general Stephani models:

- **really inhomogeneous** (not even SS) - they do not admit any spacetime symmetry at all
- the 3-dimensional hyperspaces of constant time are maximally symmetric
- the models are conformally flat (Weyl tensor $C_{abcd} = 0$)
- can be embedded into a 5-dimensional flat pseudoeuclidean space (they are embedding class one – in general any 4-dim manifold can be embedded at least locally in a 10-dim flat space)
- matter does not move along geodesics (there is non-zero acceleration $\dot{u}_a \neq 0$); models are shearfree $\sigma_{ab} = 0$
- the curvature index $k = k(t)$ changes in time so that the spatial curvature may change during evolution
- **possess the Friedmann limit** when the curvature index $k(t) \rightarrow$ const. $= 0, \pm 1$

Inhomogeneous pressure models - topology

Topology can be uncovered, if we assume the energy density to be constant, i.e.,

$$\frac{8\pi G}{c^2}\varrho = 3C_0^2 = \text{const.}\,, \tag{23}$$

$$\frac{8\pi G}{c^4}p = -3C_0^2 = \text{const.}\,, \tag{24}$$

which is essentially the **de Sitter** Universe with dark energy equation of state ($w = -1$) with global topology being $S^3 \times R$ represented by a one-sheet hyperboloid,
but with local topology of the constant time hypersurfaces (index $k(t)$) changing in time.
Usually we cut hyperboloid by either $k = 1$ (S^3 topology), $k = 0$ (R^3) or $k = -1$ (H^3).

Here we have "3-in-1" and the Universe may either **"open up" or "close down"**.

Inhomogeneous pressure models - topology

General model:

- Global topology still $S^3 \times R$. However, they are just **specific deformations of the de Sitter** hyperboloid near the "neck circle".
- The center of symmetry is **moving** around the deformed hyperboloid.
- In fact, due to a choice of the radial coordinate, there are two antipodal centers of symmetry (as in LTB model).

Inhomogeneous pressure models - singularities, EOS

- standard **Big-Bang** singularities $a \to 0$, $\varrho \to 0$, $p \to 0$ are possible (FRW limit)
- **Finite Density (FD)** singularities of pressure appear at some particular values of the spatial coordinates x, y, z (or a radial coordinate r, if in a SS model)
- **Π-boundary** - a spacelike boundary which divides each negative curvature $k(t) < 0$ section onto the two sheets (the "far sheet" and the "near-sheet")
- Π-boundary appears whenever $V(t, r) = 1 + (1/4)k(t)[(x - x_0)^2 + \ldots] = 0$
- the Universe behaves asymptotically de Sitter on a Π-boundary ($p = -\varrho$)
- There is **no global equation of state** - it changes from place to place (depends on x, y, z or r) and on the hypersurfaces $t =$ const.

FD singularities versus SFS singularities

- In inhomogeneous pressure models there are Finite Density singularities of pressure.
- In standard FRW cosmology there exist exotic (sudden future) singularities of pressure (SFS) with finite scale factor and energy density, i.e.,

$$a = \mathrm{const.}, \quad \dot{a} = \mathrm{const}, \quad \varrho = \mathrm{const}, \quad \ddot{a} \to \pm\infty, \quad p \to \mp\infty. \tag{25}$$

- They are different: FD singularities are spatial (appear somewhere in space) while SFS are temporal (appear in time on one $(t = t_s)$ of the hypersurfaces).
- There are hybrid models in which appear both FD and SFS singularities of pressure (MPD, PRD '05).

FD singularities versus SFS singularities

Such "inhomogeneized" SFS may appear in a general (no symmetry at all) inhomogeneous pressure model which can be shown by inserting the time derivative of the Stephani energy density function and the function $V(t,x,y,z)$ into the expression for the pressure, i.e.,

$$p(t,x,y,z) = -3\frac{\dot{a}^2}{a^2} - 3\frac{k}{a^2} + \frac{\dot{a}}{a}\left[2\frac{\ddot{a}}{a} - 2\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\left(\dot{k}\frac{a}{\dot{a}} - 2k\right)\right]\frac{\left[\frac{V(t,x,y,z)}{a(t)}\right]}{\left[\frac{V(t,x,y,z)}{a(t)}\right]^{\cdot}} . \quad (26)$$

It emerges that a SFS $p \to \pm\infty$ appears for $\ddot{a} \to -\infty$, if $(V/a)/(V/a)^{\cdot}$ is regular and the sign of the pressure depends on the signs of both $\dot{a}/a$ and $(V/a)/(V/a)^{\cdot}$. In fact, SF singularities appear independently of FD singularities whenever $\ddot{a} \to -\infty$ and the blow-up of p is guaranteed by the involvement of the time derivative of the function $C(t)$ in (14).

Exact inhomogeneous pressure models

I found **two explicit models** which are called **Model I and Model II** (note: time coordinate will be labeled τ instead of t and the scale factor $R(t)$ instead of $a(t)$). For the Model I we have

$$\begin{aligned} k(\tau) &= -4\frac{a}{c^2}R(\tau)\ , && (27)\\ R(\tau) &= a\tau^2 + b\tau + d\ , && (28)\\ V(\tau,r) &= 1 - \frac{a}{c^2}\left(a\tau^2 + b\tau + d\right)r^2\ , && (29)\\ \Delta &\equiv 4ad - b^2 + 1 = 0\ , && (30) \end{aligned}$$

with a, b, d = const. and for the cosmic time τ taken in sMpc/km we have: [a] = $km^2/(s^2Mpc)$, [b] = km/s and [c] = Mpc. More general models appear for $\Delta \neq 0$ - the FD pressure singularity shows up at a finite distance $r = 2/(\sqrt{-\Delta})$ (MPD '93, Barrett and Clarkson CQG 2000).

Exact inhomogeneous pressure models

For the Model II we have

$$k(\tau) = -\frac{\alpha\beta}{c^2} R(\tau) , \tag{31}$$

$$R(\tau) = \beta\tau^{\frac{2}{3}} , \tag{32}$$

$$V(\tau, r) = 1 - \frac{1}{4c^2}\alpha\beta^2\tau^{\frac{2}{3}}r^2 , \tag{33}$$

with α, β = const. with $[\alpha]$ = $(s/km)^{\frac{2}{3}}Mpc^{-\frac{4}{3}}$ and $[\beta]$ = $(km/s)^{\frac{2}{3}}Mpc^{\frac{1}{3}}$. Both models possess the Friedman limit; ($a \to 0$ for MI and $\alpha \to 0$ for MII). The common point between MI and MII is that for them $\left(\frac{k}{R}\right)_{,\tau} = 0$, where $(\ldots)_{,\tau} \equiv \frac{\partial}{\partial\tau}$.

Exact inhomogeneous pressure models

Another example of the model II (and I as well since $\beta = -4a^2/c^2$ (a = const.)) is Stelmach-Jakacka model (CQG 18, 2643 (2001)) in which one assumes that **at the center of symmetry** the standard barotropic equation of state

$$\frac{p(\tau)}{c^2} = w\varrho(\tau) \tag{34}$$

is fulfilled. For $w = 0$ one has the dust equation of state at the center, for $w = -1/3$ one has the cosmic strings. This assumption gives that

$$\frac{8\pi G}{3c^2}\varrho(\tau) = C^2(\tau) = \frac{A^2}{a^{3(w+1)}(\tau)} \quad (A = \text{const.}) \tag{35}$$

and allows to write a generalized Friedmann equation as

$$\frac{1}{c^2}\left(\frac{a_{,\tau}}{a(\tau)}\right)^2 = \frac{A^2}{a^{3(w+1)}(\tau)} - \frac{\beta}{a(\tau)} \tag{36}$$

Exact inhomogeneous pressure models

and

$$\frac{p(\tau)}{c^2} = \left[w + \frac{\beta}{4}(w+1)a(\tau)r^2 \right] \varrho(\tau) = w_{eff}\varrho(\tau) \quad . \tag{37}$$

Similarly as in the Friedmann model, we can define critical density as

$$\varrho_{cr}(\tau) = \frac{3c^2}{8\pi G} \left(\frac{a_{,\tau}}{a(\tau)} \right)^2 \tag{38}$$

and the density parameter $\Omega(\tau) = \varrho(\tau)/\varrho_{cr}(\tau)$ which after taking $\tau = \tau_0$ gives

$$1 = \frac{A^2}{H_0^2 a^{3(w+1)}(\tau_0)} - \frac{\beta c^2}{H_0^2 a_0} \equiv \Omega_0 + \Omega_{inh} \quad , \tag{39}$$

and so

$$\beta = \frac{a_0 H_0^2}{c^2} (\Omega_0 - 1) \quad , \tag{40}$$

with the unit $[\beta] = Mpc^{-1}$.

Inhomogeneous pressure models - null geodesics

The four-velocity and the acceleration for MI and MII are

$$u_\tau = -\, c\frac{1}{V}\,, \qquad \dot{u}_r = -\, c\frac{V_{,r}}{V}\,. \tag{41}$$

The components of the vector tangent to zero geodesic are

$$k^\tau = \frac{V^2}{R}\,,\;\; k^r = \pm\frac{V^2}{R^2}\sqrt{1 - \frac{h^2}{r^2}}\,,\;\; k^\theta = 0\,,\;\; k^\varphi = h\frac{V^2}{R^2 r^2}\,, \tag{42}$$

where h = const., and the plus sign in applies to a ray moving away from the centre, while the minus sign applies to a ray moving towards the centre. The acceleration scalar for MI and MII, respectively, is

$$\dot{u} \equiv (\dot{u}_a \dot{u}^a)^{\frac{1}{2}} = \frac{V_{,r}}{R} = \begin{cases} -\,2\frac{a}{c^2}r, \\ -\,\frac{1}{2}\alpha\beta r, \end{cases} \tag{43}$$

and it does not depend on the time coordinate at all.

Inhomogeneous pressure models - redshift

The point:

The further away from the center $r = 0$ is an observer, the larger acceleration he subjects.

The redshift is given by (for MI and MII, respectively)

$$1 + z = \frac{(u_a k^a)_G}{(u_a k^a)_O} = \begin{cases} \dfrac{\left[\dfrac{1 - \frac{a}{c^2}(a\tau^2 + b\tau + d)r^2}{a\tau^2 + b\tau + d}\right]_G}{\left[\dfrac{1 - \frac{a}{c^2}(a\tau^2 + b\tau + d)r^2}{a\tau^2 + b\tau + d}\right]_O}, \\ \dfrac{\left[\dfrac{1 - \frac{1}{4}\alpha\beta^2\tau^{\frac{2}{3}}r^2}{\beta\tau^{\frac{2}{3}}}\right]_G}{\left[\dfrac{1 - \frac{1}{4}\alpha\beta^2\tau^{\frac{2}{3}}r^2}{\beta\tau^{\frac{2}{3}}}\right]_O}. \end{cases} \tag{44}$$

Inhomogeneous pressure models - redshift

The constant h and the angle ϕ between the direction of observation and the direction defined by the observer and the center of symmetry are related by

$$cos\phi = \pm\sqrt{1 - \frac{h^2}{r^2}} \; . \tag{45}$$

4. Inhomogeneous pressure models - luminosity distance.

Taking complexity of models into account, it is best to apply the series expansion of the redshift-magnitude formula (Kristian and Sachs 1966) given by (calculated to higher-orders in MPD & Stachowiak '06)

$$m_{bol} = M - 5\log_{10}\left(u_{a;b}K^a K^b\right)_O + 5\log_{10} cz$$
$$+\frac{5}{2}\left(\log_{10} e\right)\left\{\left(4 - \frac{u_{a;bc}K^a K^b K^c}{\left(u_{a;b}K^a K^b\right)^2}\right) z + \mathbf{O}\left(z^2\right)\right\}_O \ , \qquad (46)$$

where

$$u_{a;b} = \frac{1}{3}\Theta h_{ab} - \dot{u}_a u_b \ , \quad h_{ab} \equiv g_{ab} + u_a u_b \ , \quad K^a \equiv \frac{k^a}{u_b k^b} \ , \quad u_a u^a = -1 \ . \qquad (47)$$

The projection of K^a onto the spatial hypersurfaces ortogonal to u_a is a spatial unit vector pointing in the observer direction

$$n^a = -u^a - K^a, \quad n^a n_a = 1 \ . \qquad (48)$$

Inhomogeneous pressure models - MI central observations

After some algebra we find redshift-magnitude relations for the Model I as follows (the only inhomogeneous pressure parameter is a, since we have chosen $b = 1$ and $d = 0$ without loosing a generality)

$$m = M + 25 + 5\log_{10}\left[cz\left(\frac{a\tau_0^2+\tau_0}{2a\tau_0+1}\right)\right] + 1.086\left[1 + 4a\frac{\left(a\tau_0^2+\tau_0\right)}{\left(2a\tau+1\right)^2}\right]z. \quad (49)$$

This relation has no difference with the FRW relation

$$m_{bol} = M - 5\log_{10}\tilde{H}_0 + 5\log_{10}cz + 1.086\left(1-\tilde{q}_0\right)z, \quad (50)$$

provided we have defined the Hubble and deceleration parameters as

$$\tilde{H}_0 = \frac{2a\tau_0+1}{a\tau_0^2+\tau_0}, \qquad \tilde{q}_0 = -4a\frac{a\tau_0^2+\tau_0}{(2a\tau_0+1)^2}, \quad (51)$$

and they may be taken with the same values as in FRW models.

Inhomogeneous pressure models - MII central observations.

For the Model II we have

$$m_{bol} = M - 5\log_{10}\tilde{H}_0 + 5\log_{10} cz + 1.086\,(1-\tilde{q}_0)\,z, \qquad (52)$$

where

$$\tilde{H}_0 = \frac{2}{3\tau_0}, \qquad \tilde{q}_0 = \frac{1}{2} - \frac{9}{8}c^2\alpha\tau_0^{\frac{4}{3}}. \qquad (53)$$

are the Friedman values of the Hubble and the deceleration parameters.

Inhomogeneous pressure models against supernovae data

In (MPD + Hendry '98) we first compared with Perlmutter P97 (Ap.J. 483, 565 (1997)) data which was in favor of deceleration $(a < 0)$, but the advantage was that inhomogeneous pressure models gave a **longer age** of the Universe.

According to the current SnIa data (77 supernovae of Riess eta al. for z <0.5) we have the best fit values of inhomogeneity parameter a of the Model I to be

$$a = 10^6 km^2/s^2 Mpc > 0. \tag{54}$$

Inhomogeneous pressure models against supernovae data

Godłowski, Stelmach and Szydłowski (CQG 21, 3953 (2004)) checked Stelmach-Jakacka (model II) which had approximate dust equation of state $p = 0$ at the center of symmetry $r = 0$.

Their results showed that

$$\Omega_{inh,0} = 0.61^{+0.08}_{-0,10} \tag{55}$$

so that the inhomogeneity can mimic the dark energy.

The inhomogeneity has dominated the universe quite recently so it influences only slightly the Doppler peaks (move them to larger l's) and does not influence BBN nucleosynthesis at all since then it is just negligible.

Models of type I of arbitrary a, b and d have earlier been studied by Barrett and Clarkson (CQG 24, 5047 (2000)) and they also showed the model may fit to the observational data.

Inhomogeneous pressure - accelerated away observers

Since the acceleration scalar is

$$\dot{u} = -2\frac{a}{c^2}r \quad , \tag{56}$$

with r being the radial coordinate of the model, then

the high pressure region is at $r = 0$ (center of symmetry), while the low (negative) pressure regions are outside the center, so that the particles are **accelerated away from the center**

which is a similar effect to that caused by the positive cosmological constant in ΛCDM model.

The difference is that in ΛCDM the pressure is constant everywhere while in Stephani models it depends on the spatial coordinates.

Inhomogeneous pressure models - MII non-central observations

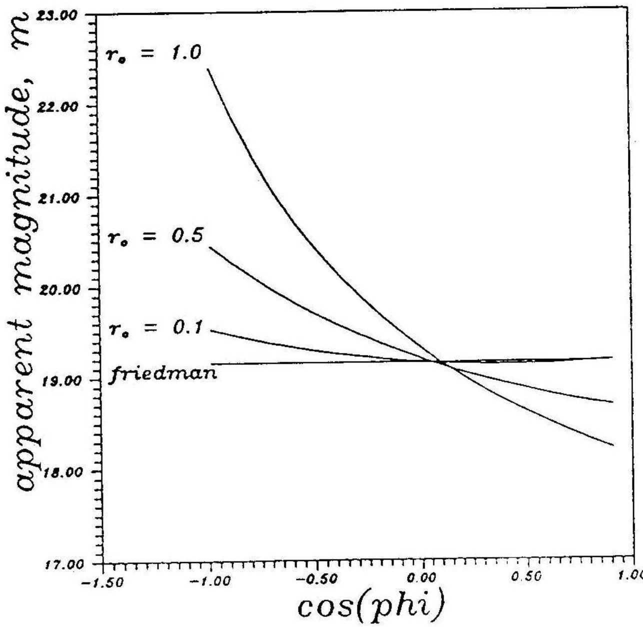
It is most challenge to actually compare inhomogeneous pressure model with supernovae data **for non-centrally placed observers**, i.e., us being slightly away from the center of symmetry. For the Stephani Model II the redshift-magnitude relation in series expansion gets a bit complicated (here $r \neq 0$, and $h \neq 0$ in the previous formulas)

$$m_{bol} = M + 25 + 5\log_{10}\left[\frac{cz}{\frac{2}{3}\frac{1}{\tau_0} + \frac{1}{2}c\alpha\beta r_0\cos\phi}\right] + 1.086z \times$$

$$\left[\frac{\frac{2}{9}\frac{1}{\tau_0^2}\left(1 + \frac{3}{4}\alpha\beta^2\tau_0^{\frac{2}{3}}r_0^2\right) - c\alpha\beta\frac{r_0}{\tau_0}\cos\phi + \frac{1}{2}c^2\alpha\tau_0^{-\frac{2}{3}}\left(1 - \frac{5}{4}\alpha\beta^2\tau_0^{\frac{2}{3}}r_0^2\right)\cos^2\phi}{\left(\frac{2}{3}\frac{1}{\tau_0} + \frac{1}{2}c\alpha\beta r_0\cos\phi\right)^2}\right].$$

Inhomogeneous pressure (observations) - non-centrally placed observers

Here we are **an example theoretical plot** for non-centrally placed observers



A plot of the dependence of the apparent magnitude on the direction in the sky for the model MII. We fix the redshift of a galaxy to be $z = 0.5$ and $\alpha c^2 = 100(km/sMpc)^{-\frac{4}{3}}$, $\beta = 1.1 \cdot 10^5 (km/s)^{\frac{2}{3}} Mpc^{\frac{1}{3}}$, $\tau_0^{-1} = 75 km/(sMpc) - 1 < \cos\phi < 1$ and $r_0 = 0.1, 0.5, 1.0$.

5. Redshift drift in inhomogeneous pressure models.

Let us assume that that the source does not posses any peculiar velocity, so it maintains a fixed comoving coordinate $dr = 0$. The light emitted by the source at two different times τ_e and $\tau_e + \delta\tau_e$ will be observed at τ_o and $\tau_o + \delta\tau_o$ related by

$$\int_{\tau_e}^{\tau_o} \frac{d\tau}{a(\tau)} = \int_{\tau_e+\delta\tau_e}^{\tau_o+\delta\tau_o} \frac{d\tau}{a(\tau)} \ . \qquad (57)$$

For small $\delta\tau_e$ and $\delta\tau_o$ we have

$$\frac{\delta\tau_e}{a(\tau_e)} = \frac{\delta\tau_o}{a(\tau_o)} \ . \qquad (58)$$

Redshift drift in inhomogeneous pressure models.

For small $\delta\tau_e$ and $\delta\tau_o$ we have

$$\begin{aligned}
(u_a k^a)_o &= (u_a k^a)(r_0, \tau_0 + \delta\tau_0) = (u_a k^a)(r_0, \tau_0) + \left[\frac{\partial(u_a k^a)}{\partial\tau}\right]_{(r_0,\tau_0)} \delta\tau_0 \\
(u_a k^a)_e &= (u_a k^a)(r_e, \tau_e + \delta\tau_e) = (u_a k^a)(r_e, \tau_e) + \left[\frac{\partial(u_a k^a)}{\partial\tau}\right]_{(r_e,\tau_e)} \delta\tau_e \; ,
\end{aligned}$$

where for our models

$$u_a k^a = -\frac{1 + \frac{1}{4}k(\tau)r^2}{a(\tau)} \; . \tag{59}$$

From the definition of the redshift drift formula (Sandage 1962) we have

$$\delta z = \frac{(u_a k^a)(r_e, \tau_e + \delta\tau_e)}{(u_a k^a)(r_0, \tau_0 + \delta\tau_0)} - \frac{(u_a k^a)(r_e, \tau_e)}{(u_a k^a)(r_0, \tau_0)} \; , \tag{60}$$

Redshift drift in inhomogeneous pressure models.

Then, we obtain

$$\frac{\delta z}{\delta \tau} = -\frac{H_0}{1+\frac{1}{4}k(\tau_0)r_0^2}\left[\frac{H}{H_0} - (1+z)\right] , \qquad (61)$$

which with the help of the definitions of the model's parameters Ω_0 and Ω_{inh} can be rewritten in the following form

$$\frac{\delta z}{\delta \tau} = -\frac{H_0}{1+\frac{1}{4}H_0^2(\Omega_0-1)\tilde{r}_0^2}\left[\sqrt{\Omega_0\tilde{a}^{-3(w+1)} + (1-\Omega_0)\tilde{a}^{-1}} - (1+z)\right] , \qquad (62)$$

where $\tilde{a} = \frac{a}{a_0}$ and $\tilde{r} = ra_0$.

Redshift drift in inhomogeneous pressure models.

Eventually we end up with the following set of formulas that combined together allows us to find the rate of change of redshift $\frac{\delta z}{\delta \tau}$ (**a redshift drift**) of any source at redshift z in the considered class of Stephani model defined by the relation $k(\tau) = \beta a(\tau)$.

$$\frac{\delta z}{\delta \tau} = -\frac{H_0}{1 + \frac{1}{4} H_0^2 (\Omega_0 - 1)\tilde{r}_0^2} \left(\sqrt{\Omega_0 \tilde{a}^{-3(w+1)} + (1 - \Omega_0)\tilde{a}^{-1}} - 1 - z \right) \tag{63}$$

$$\tilde{a}^{-1} = \frac{1 + \frac{H_0^2}{4}(\Omega_0 - 1)\tilde{r}_0^2}{1 + \frac{H_0^2}{4}(\Omega_0 - 1)\tilde{a}\tilde{r}^2}(1 + z), \tag{64}$$

$$\frac{d\tilde{r}}{d\tau} = \pm \tilde{a}^{-1} \left(1 - \frac{\tilde{r}_0^2}{\tilde{r}^2} sin^2 \phi \right)^{1/2}. \tag{65}$$

where the last equation describes the propagation of the null geodesic.

Redshift drift in inhomogeneous pressure models.

In the limit where $\Omega_0 = 1 \Rightarrow \Omega_{inh} = 0$ and $w = 0$ (a flat FRW model filled with dust) the formula (63) reduces to

$$\frac{\delta z}{\delta \tau} = -H_0\left[(1+z)^{3/2} - (1+z)\right], \qquad (66)$$

which coincides with the formulas obtained in different papers investigating the problem (see for example Loeb 1998).
For the computational convenience we transform the above formulas to

$$\frac{\delta z}{\delta \tau} = -\frac{H_0}{1+\frac{1}{4}H_0^2(\Omega_0-1)\tilde{r}_0^2}\left[\sqrt{\Omega_0\tilde{a}^{-3(w+1)} + (1-\Omega_0)\tilde{a}^{-1}} - (1+z)\right] \qquad (67)$$

$$\tilde{a}^{-1} = \left[1+\frac{H_0^2}{4}(\Omega_0-1)\tilde{r}_0^2\right](1+z) - \frac{H_0^2}{4}(\Omega_0-1)\tilde{r}^2, \qquad (68)$$

$$\frac{d\tilde{r}}{dz} = \frac{1+\frac{H_0^2}{4}(\Omega_0-1)\tilde{r}_0^2}{H_0\sqrt{\frac{\Omega_0\tilde{a}^{-3(w+1)}+(1-\Omega_0)\tilde{a}^{-1}}{1-\frac{\tilde{r}_0^2}{\tilde{r}^2}sin^2\phi}} + \frac{H_0^2}{2}(\Omega_0-1)\tilde{r}}, \qquad (69)$$

Redshift drift in inhomogeneous pressure models.

(Quercellini et. all 2012) Redshift drift as a function of redshift for ΛCDM, the DGP model, the matter dominated model (CMD) and the 3 different void models (LTB). In the dark energy models (ΛCDM, DGP) the redshift drift is positive at small redshift, but becomes negative for $z \gtrsim 2$. On the other hand, a giant void mimicking dark energy produces a very distinct z dependence of the drift with $\frac{\delta z}{\delta t}$ being always negative.

Redshift drift as a function of redshift for the Stephani model with $r_0 = 0$, $w = 0$ and $\Omega_{inh} = 1 - \Omega_0 = 0.61$. The redshift drift exhibits behavior similar to the behavior of the redshift drift in the void models. However, unlike in the void models, the redshift drift becomes positive for $z \in (0, 0.34)$ and attains its highest value of order $10^{-13}/year$ for $z \sim 0.17$.

6. Conclusions

- Observations from one point in the Universe **suggest its isotropy**, but not necessarily homogeneity. This gives **motivation** for studying spherically symmetric models of the Universe.
- Two specific models have been proposed: the **Lemaître-Tolman-Bondi** model (inhomogeneous density) and the **Stephani** model (inhomogeneous pressure).
- These models have been preliminary **checked against astronomical data** which shows that the **inhomogeneities may drive acceleration** – **back-reaction of inhomogeneities**.
- There is an open question whether we **really live in** a homogeneous and isotropic (FRW) universe or at least in an isotropic (spherically symmetric) **void or an interior of an inhomogeneous pressure "exotic star"**. Especially, it is interesting to check data for non-centrally placed observers.

conclusions contd.

- Inhomogeneous pressure models have another advantage - they can even model a total spacetime inhomogeneity.
- Another advantage is that they **admit cosmic acceleration in a natural way** and can serve as dark energy.
- The formula for redshift drift of any source at redshift z in the specific class of Stephani models has been obtained.
- In the class of Stephani models considered (with a centrally placed observer) there is a subset of observationally viable models exhibiting a qualitatively different behavior of redshift drift than the LTB void models.
- This difference may allow to test inhomogeneous pressure (Stephani) models against LTB void (and other homogeneous and inhomogeneous) in future experiments aimed to measure the redshift drift.