Testing spherically symmetric inhomogeneous pressure cosmology

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Content:

- 1. Universe symmetries. Acceleration as back-reaction of inhomogeneities.
- 2. Complementary models of the spherically symmetric Universe.
- 3. Inhomogeneous pressure models - properties.
- 4. Redshift drift in inhomogeneous pressure models.
- 5. Other tests: luminosity distance, baryon acoustic oscillations, shift parameter.
- 6. Off-center observers and supernovae.
- 7. Conclusions

1. Universe symmetries. Acceleration as back-reaction of inhomogeneities.

In the context of dark energy problem (Λ being 120 orders of magnitude too large) there has been more interest in the non-friedmannian models of the universe which could explain the acceleration only due to inhomogeneity (initially E. Kolb). One of the strogest claims was that

we are living in a spherically symmetric void of density described by the Lemaître-Tolman-Bondi dust spheres model

J. Uzan, R. Clarkson, G.F.R. Ellis (PRL, **100**, 191303 (2008))

R.R. Caldwell and A. Stebbins (PRL, **100**, 191302 (2008))

C. Clarkson, B. Bassett and T. H-Ch. Lu (PRL, **101**, 011301 (2008))

and many others

In fact, there are **two ways** to get large-scale structure in cosmology:

perturb FRW models $\leftrightarrow$ **consider exact inhomogeneous models**

How symmetric is the universe?

- Einstein equations are **complicated** and to solve them we just **assume symmetries** (Occam's razor - if we play with simple symmetric models observationally, we do not need to bother about any more complicated ones).
- Why not to **paradigm** this by a fundamental principle - the **Copernican Principle** that we do not live in the center of the Universe (we really do not want to be special in the Universe).
- However, so far observations have been made just **from one point** in the Universe and extend only onto the one (and unique) past light cone.
- Even **CMB** we observe from one point - this **proves isotropy**, but not necessarily homogeneity (isotropy with respect to any point in the Universe).

Center of the Universe?

- Is the universe homogeneous?
- Suppose we have an inhomogeneous model of the Universe with the **same (small) number of parameters** as a homogeneous dark energy model and they both fit observations very well.
- Could we **differentiate** between these two models?
- Simplest inhomogeneous models are **spherically symmetric** (isotropic with respect to just one point).

New paradigm of inhomogeneity - LTB void.

- In fact, even if we restrict ourselves to spherical symmetry then there are **two complementary models** of the universe and they can both mimic homogeneous dark energy models!
- **These are: the inhomogeneous density (dust shells) Lemaître-Tolman-Bondi (LTB) models and inhomogeneous pressure (gradient of pressure shells) Stephani models.**
- Apparently **most of the researchers investigate the former** and only a few investigate the latter.
- It seems that we are about to create an LTB void paradigm of inhomogeneous density spherically symmetric dust universe

New paradigm of inhomogeneity?

- I suggest investigating **at least a complement of LTB** - spherically symmetric Stephani model of pressure gradient which also possesses a generalization which is totally spacetime inhomogeneous.
- In fact MPD and M. Hendry (Ap.J. '98) first compared an inhomogeneous model of the Universe with real observational data (SN'97 sample) from supernovae and showed that they can be fitted.
- Despite inhomogeneous density (LTB) models were theoretically explored before (since Lemaître - 1933) only later they were tested observationally against supernovae (e.g. K. Tomita, Prog. Theor. Phys. 106, 929 (2001); K. Bolejko, astro-ph/0512103).
- And there are lots of less symmetric or purely inhomogeneous models (Goode (1986), Szafron (1977), Szekeres (1975), Wainwright-Goode (1980), Ruiz-Senovilla (1992) etc.) to investigate as candidates for dark energy. See e.g. M.-N. Célérier (ArXiv:1206.6026).

2. Complementary models of the spherically symmetric Universe

Let us consider **advantages of the simplest inhomogeneous models** and show that they may fit observations, so that they are a good candidate for explanation of cosmic acceleration by an inhomogeneity.

In order to make a **complementary analysis** with LTB models the following table proves useful:

	pressure	density
FRW	$p = p(t)$	$\varrho = \varrho(t)$
LTB	$p = 0 \; (p(t))$	$\varrho = \varrho(t, r)$ - nonuniform
Stephani	$p = p(t, r)$ - nonuniform	$\varrho = \varrho(t)$

SS Lemaître-Tolman-Bondi Universe

– is the only spherically symmetric solution of Einstein equations for **pressureless matter** ($T^{ab} = \varrho u^a u^b$) and no cosmological term (G. Lemaître, Ann. Soc. Sci. Brux. **A 53**, 51 (1933); R.C. Tolman, Proc. Natl. Acad. Sci., **20**, 169 (1934); H. Bondi MNRAS **107**, 410 (1947))

$$ds^2 = -dt^2 + \frac{R'^2}{1-K} dr^2 + R^2 (d\theta^2 + sin^2\theta d\phi^2) \ , \tag{1}$$

where

$$R = R(t,r); \qquad R' = \partial R/\partial r; \qquad K = K(r) \ . \tag{2}$$

The Einstein equations reduce to

$$\dot{R}^2 = \frac{2M(r)}{R} - K(r); \qquad 2M' = \kappa \varrho R^2 R' \ , \tag{3}$$

and are solved by

SS Lemaître-Tolman-Bondi Universe

$$R(r,\eta) = \frac{M(r)}{K(r)}\Phi'(\eta); \qquad t(r,\eta) = T_0(r) + \frac{M(r)}{K^{3/2}(r)}\phi'(\eta) \ , \tag{4}$$

where for $K(r) < 0$ (hyperbolic), $K(r) = 0$ (parabolic), and $K(r) > 0$ (elliptic) appropriately ($K(r)$ is a spatially dependent "curvature index") we have

$$\Phi(\eta) = (\sinh\eta - \eta; \eta^3/6; \eta - \sin\eta) \ . \tag{5}$$

Regularity conditions:

- existence of a regular **center of symmetry** $r = 0$ – implies $R(t,0) = \dot{R}(t,0) = 0$ and $M(0) = M'(0) = K(0) = K'(0) = 0$ and $R' \to 1$.
- hypersurfaces of constant time are **orthogonal** to 4-velocity and are of topology S^3 – implies the existence of a second center of symmetry $r = r_c$ (with some ‘turning value’ $0 < r_{tv} < r_c$)
- a ‘shell-crossing’ singularity should be **avoided** – implies $R'(t,r) \neq 0$ except at turning values (though it is a weak singularity - no geodesic incompleteness)

SS Lemaître-Tolman-Bondi Universe

Kinematic characteristics of the model:

$$u_{a;b} = \frac{1}{3}\Theta h_{ab} + \sigma_{ab} \quad , \tag{6}$$

Expansion scalar:

$$\Theta = \frac{2\dot{R}}{R} + \frac{\dot{R}'}{R'} \, , \tag{7}$$

Shear tensor and scalar:

$$\sigma^{ab} = \Sigma\zeta^{ab}; \qquad \zeta^{ab} \equiv h^{ab} - 3v^a v^b; \tag{8}$$

$$\Sigma = \frac{1}{6}\sigma_{ab}\zeta^{ab} = -\frac{1}{3}\left(\frac{\dot{R}'}{R'} - \frac{\dot{R}}{R}\right) \quad , \tag{9}$$

and $v^a = \sqrt{h^{rr}}\delta^a_r$ is the unit vector orthogonal to u^a and to the 2-sphere orbits of $SO(3)$

In LTB models a Big-Bang is not necessarily instantenuous - different points start at different moments.

Friedmann limit is obtained for:

$$R(t,r) = a(t)r; \qquad M(r) = M_0 r^3; \qquad K(r) = k_0 r^2 \ , \tag{10}$$

SS Stephani Universe

– is the only spherically symmetric solution of Einstein equations for perfect-fluid energy-momentum tensor ($T^{ab} = (\varrho + p)u^a u^b + pg^{ab}$) which is **conformally flat** and **embeddable** in a 5-dimensional flat space (H. Stephani Commun. Math. Phys. **4**, 167 (1967); A. Krasiński, GRG **15**, 673 (1983)). After introducing a Friedmann-like time coordinate (cf. later) we have

$$ds^2 = -\frac{a^2}{\dot{a}^2}\frac{a^2}{V^2}\left[\left(\frac{V}{a}\right)^{\cdot}\right]^2 dt^2 + \frac{a^2}{V^2}\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] , \quad (11)$$

where

$$V(t,r) = 1 + \frac{1}{4}k(t)r^2 , \quad (12)$$

and $(\ldots)^{\cdot} \equiv \partial/\partial t$. The function $a(t)$ plays the role of a **generalized scale factor**, $k(t)$ has the meaning of **a time-dependent "curvature index"**, and r is the radial coordinate.

SS Stephani Universe

The energy density and pressure are given by

$$\varrho(t) = 3\left[\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k(t)}{a^2(t)}\right] , \tag{13}$$

$$p(t,r) = \varrho(t)\left\{-1 + \frac{1}{3}\frac{\dot{\varrho}(t)}{\varrho(t)}\frac{\left[\frac{V(t,r)}{a(t)}\right]^{\cdot}}{\left[\frac{V(t,r)}{a(t)}\right]}\right\} \equiv w_{eff}(t,r)\varrho(t) , \tag{14}$$

and generalize the standard Einstein-Friedmann relations

$$\varrho(t) = 3\left(\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k}{a^2(t)}\right) , \tag{15}$$

$$p(t) = -\left(2\frac{\ddot{a}(t)}{a(t)} + \frac{\dot{a}^2(t)}{a^2(t)} + \frac{k}{a^2(t)}\right) \tag{16}$$

to inhomogeneous models.

SS Stephani Universe

Kinematic characteristic of the model:

$$u_{a;b} = \frac{1}{3}\Theta h_{ab} - \dot{u}_a u_b \ , \qquad \dot{u} \equiv (\dot{u}_a \dot{u}^a)^{\frac{1}{2}} \ . \tag{17}$$

where $\dot{u}$ is the acceleration scalar and the acceleration vector

$$\dot{u}_r = \frac{\left\{ \frac{a^2}{\dot{a}^2} \frac{a^2}{V^2} \left[\left(\frac{V}{a} \right)^{\cdot} \right] \right\}_{,r}}{\frac{a^2}{\dot{a}^2} \frac{a^2}{V^2} \left[\left(\frac{V}{a} \right)^{\cdot} \right]} \tag{18}$$

while the expansion scalar is the same as in FRW model, i.e.,

$$\Theta = 3\frac{\dot{a}}{a} \ . \tag{19}$$

3. Fully inhomogeneous pressure models - properties.

The general Stephani metric reads as

$$ds^2 = -\frac{a^2}{\dot{a}^2}\frac{a^2}{V^2}\left[\left(\frac{V}{a}\right)^{\cdot}\right]^2 dt^2 + \frac{a^2}{V^2}\left[dx^2 + dy^2 + dz^2\right], \quad (20)$$

$$V(t,x,y,z) = 1 + \frac{1}{4}k(t)\left\{[x - x_0(t)]^2 + [y - y_0(t)]^2 + [z - z_0(t)]^2\right\},$$

and x_0, y_0, z_0 are arbitrary functions of time. This is just a generalization of the FRW metric in isotropic coordinates

$$ds^2 = -dt^2 + \frac{a^2(t)}{1 + \frac{1}{4}kr^2}\left(dr^2 + r^2 d\Omega^2\right); \qquad r^2 = x^2 + y^2 + z^2 \quad (21)$$

which by a transformation $\bar{r} = 1 + (1/2)kr^2$ can be brought to a standard form

$$d\bar{s}^2 = -dt^2 + a^2(t)\left(\frac{d\bar{r}^2}{1 - k\bar{r}^2} + \bar{r}^2 d\Omega^2\right). \quad (22)$$

Inhomogeneous pressure models - properties

Properties of general Stephani models:

- **really inhomogeneous** (not even SS) - they do not admit any spacetime symmetry at all
- the 3-dimensional hyperspaces of constant time are maximally symmetric
- the models are conformally flat (Weyl tensor $C_{abcd} = 0$)
- can be embedded into a 5-dimensional flat pseudoeuclidean space (they are embedding class one – in general any 4-dim manifold can be embedded at least locally in a 10-dim flat space)
- matter does not move along geodesics (there is non-zero acceleration $\dot{u}_a \neq 0$); models are shearfree $\sigma_{ab} = 0$
- the curvature index $k = k(t)$ changes in time so that the spatial curvature may change during evolution
- **possess the Friedmann limit** when the curvature index $k(t) \to$ const. $= 0, \pm 1$

Inhomogeneous pressure models - topology

Topology can be uncovered, if we assume the energy density to be constant, i.e.,

$$\frac{8\pi G}{c^2}\varrho = 3C_0^2 = \text{const.}\,, \tag{23}$$

$$\frac{8\pi G}{c^4}p = -3C_0^2 = \text{const.}\,, \tag{24}$$

which is essentially the **de Sitter** Universe with dark energy equation of state ($w = -1$) with global topology being $S^3 \times R$ represented by a one-sheet hyperboloid,

but with local topology of the constant time hypersurfaces (index $k(t)$) changing in time.

Usually we cut hyperboloid by either $k = 1$ (S^3 topology), $k = 0$ (R^3) or $k = -1$ (H^3).

Here we have "3-in-1" and the Universe may either **"open up" or "close down"**.

Inhomogeneous pressure models - topology

General model:

- Global topology still $S^3 \times R$. However, they are just **specific deformations of the de Sitter** hyperboloid near the "neck circle".
- The center of symmetry is **moving** around the deformed hyperboloid.
- In fact, due to a choice of the radial coordinate, there are two antipodal centers of symmetry (as in LTB model).

Inhomogeneous pressure models - singularities, EOS

- standard **Big-Bang** singularities $a \to 0$, $\varrho \to 0$, $p \to 0$ are possible (FRW limit)
- **Finite Density (FD)** singularities of pressure appear at some particular values of the spatial coordinates x, y, z (or a radial coordinate r, if in a SS model)
- **Π-boundary** - a spacelike boundary which divides each negative curvature $k(t) < 0$ section onto the two sheets (the "far sheet" and the "near-sheet")
- Π-boundary appears whenever $V(t, r) = 1 + (1/4)k(t)[(x - x_0)^2 + \ldots] = 0$
- the Universe behaves asymptotically de Sitter on a Π-boundary ($p = -\varrho$)
- There is **no global equation of state** - it changes from place to place (depends on x, y, z or r) and on the hypersurfaces $t =$ const.

FD singularities versus SFS singularities

- In inhomogeneous pressure models there are Finite Density singularities of pressure.
- In standard FRW cosmology there exist exotic (sudden future) singularities of pressure (SFS) with finite scale factor and energy density, i.e.,

$$a = \text{const.}, \quad \dot{a} = \text{const}, \quad \varrho = \text{const}, \quad \ddot{a} \rightarrow \pm\infty, \quad p \rightarrow \mp\infty. \tag{25}$$

- They are different: FD singularities are spatial (appear somewhere in space) while SFS are temporal (appear in time on one $(t = t_s)$ of the hypersurfaces).
- There are hybrid models in which appear both FD and SFS singularities of pressure (MPD, PRD ’05).

FD singularities versus SFS singularities

Such “inhomogeneized” SFS may appear in a general (no symmetry at all) inhomogeneous pressure model which can be shown by inserting the time derivative of the Stephani energy density function and the function $V(t,x,y,z)$ into the expression for the pressure, i.e.,

$$p(t,x,y,z) = -3\frac{\dot{a}^2}{a^2} - 3\frac{k}{a^2} + \frac{\dot{a}}{a}\left[2\frac{\ddot{a}}{a} - 2\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\left(\dot{k}\frac{a}{\dot{a}} - 2k\right)\right]\frac{\left[\frac{V(t,x,y,z)}{a(t)}\right]}{\left[\frac{V(t,x,y,z)}{a(t)}\right]^{\cdot}} . \tag{26}$$

It emerges that a SFS $p \to \pm\infty$ appears for $\ddot{a} \to -\infty$, if $(V/a)/(V/a)^{\cdot}$ is regular and the sign of the pressure depends on the signs of both $\dot{a}/a$ and $(V/a)/(V/a)^{\cdot}$. In fact, SF singularities appear independently of FD singularities whenever $\ddot{a} \to -\infty$ and the blow-up of p is guaranteed by the involvement of the time derivative of the function $\rho(t)$ in (14).

Exact inhomogeneous pressure models

I found **two explicit models** which are called **Model I and Model II** (note: time coordinate will be labeled τ instead of t and the scale factor $R(t)$ instead of $a(t)$).
For the Model I we have

$$
\begin{aligned}
k(\tau) &= -4\frac{a}{c^2}R(\tau)\,, && (27)\\
R(\tau) &= a\tau^2 + b\tau + d\,, && (28)\\
V(\tau,r) &= 1 - \frac{a}{c^2}\left(a\tau^2 + b\tau + d\right)r^2\,, && (29)\\
\Delta &\equiv 4ad - b^2 + 1 = 0\,, && (30)
\end{aligned}
$$

with a, b, d = const. and for the cosmic time τ taken in sMpc/km we have: [a] = $km^2/(s^2 Mpc)$, [b] = km/s and [c] = Mpc. More general models appear for $\Delta \neq 0$ - the FD pressure singularity shows up at a finite distance $r = 2/(\sqrt{-\Delta})$ (MPD '93, Barrett and Clarkson CQG 2000).

Exact inhomogeneous pressure models

For the Model II we have

$$k(\tau) = -\frac{\alpha\beta}{c^2}R(\tau)\ , \tag{31}$$

$$R(\tau) = \beta\tau^{\frac{2}{3}}\ , \tag{32}$$

$$V(\tau,r) = 1 - \frac{1}{4c^2}\alpha\beta^2\tau^{\frac{2}{3}}r^2\ , \tag{33}$$

with α, β = const. with $[\alpha]$ = $(s/km)^{\frac{2}{3}}Mpc^{-\frac{4}{3}}$ and $[\beta]$ = $(km/s)^{\frac{2}{3}}Mpc^{\frac{1}{3}}$. Both models possess the Friedman limit; ($a \to 0$ for MI and $\alpha \to 0$ for MII). The common point between MI and MII is that for them $\left(\frac{k}{R}\right)_{,\tau} = 0$, where $(\ldots)_{,\tau} \equiv \frac{\partial}{\partial\tau}$.

Exact inhomogeneous pressure models

Another example of the model II (and I as well since $\beta = -4a^2/c^2$ (a = const.)) is Stelmach-Jakacka model (CQG 18, 2643 (2001)) in which one assumes that **at the center of symmetry** the standard barotropic equation of state

$$\frac{p(\tau)}{c^2} = w\varrho(\tau) \tag{34}$$

is fulfilled. For $w = 0$ one has the dust equation of state at the center, for $w = -1/3$ one has the cosmic strings. This assumption gives that

$$\frac{8\pi G}{3c^2}\varrho(\tau) = C^2(\tau) = \frac{A^2}{a^{3(w+1)}(\tau)} \quad (A = \text{const.}) \tag{35}$$

and allows to write a generalized Friedmann equation as

$$\frac{1}{c^2}\left(\frac{a_{,\tau}}{a(\tau)}\right)^2 = \frac{A^2}{a^{3(w+1)}(\tau)} - \frac{\beta}{a(\tau)} \tag{36}$$

Exact inhomogeneous pressure models

and

$$\frac{p(\tau)}{c^2} = \left[w + \frac{\beta}{4}(w+1)a(\tau)r^2\right]\varrho(\tau) = w_{eff}\,\varrho(\tau) \quad . \tag{37}$$

Similarly as in the Friedmann model, we can define critical density as

$$\varrho_{cr}(\tau) = \frac{3c^2}{8\pi G}\left(\frac{a_{,\tau}}{a(\tau)}\right)^2 \tag{38}$$

and the density parameter $\Omega(\tau) = \varrho(\tau)/\varrho_{cr}(\tau)$ which after taking $\tau = \tau_0$ gives

$$1 = \frac{A^2}{H_0^2 a^{3(w+1)}(\tau_0)} - \frac{\beta c^2}{H_0^2 a_0} \equiv \Omega_0 + \Omega_{inh} \quad , \tag{39}$$

and so

$$\beta = \frac{a_0 H_0^2}{c^2}\left(\Omega_0 - 1\right) \quad , \tag{40}$$

with the unit $[\beta] = Mpc^{-1}$.

Inhomogeneous pressure models - null geodesics

The four-velocity and the acceleration for MI and MII are

$$u_\tau = -\,c\frac{1}{V}\,, \qquad \dot{u}_r = -\,c\frac{V_{,r}}{V}\,. \tag{41}$$

The components of the vector tangent to zero geodesic are

$$k^\tau = \frac{V^2}{R}\,, \quad k^r = \pm\frac{V^2}{R^2}\sqrt{1\,-\,\frac{h^2}{r^2}}\,, \quad k^\theta = 0\,, \quad k^\varphi = h\frac{V^2}{R^2r^2}\,, \tag{42}$$

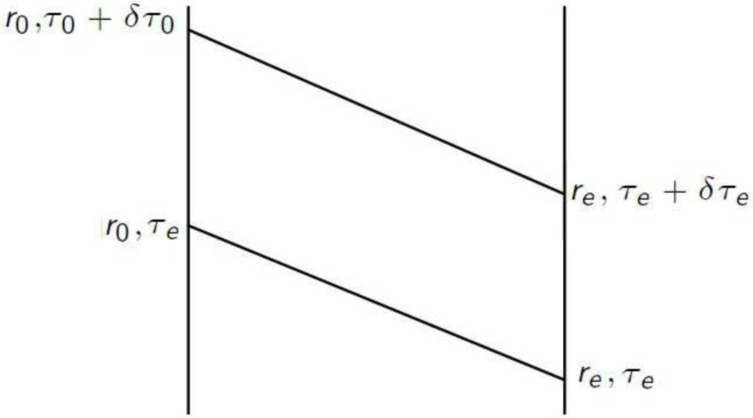
where h = const., and the plus sign in applies to a ray moving away from the centre, while the minus sign applies to a ray moving towards the centre. The acceleration scalar for MI and MII, respectively, is

$$\dot{u}\;\equiv\left(\dot{u}_a\dot{u}^a\right)^{\frac{1}{2}}\;=\;\frac{V_{,r}}{R}\;=\;\begin{cases}-\,2\frac{a}{c^2}r,\\ -\,\frac{1}{2}\alpha\beta r,\end{cases} \tag{43}$$

The farther away from the center at $r = 0$, the larger (negative) acceleration.

4. Redshift drift in inhomogeneous pressure models.

Redshift drift (Sandage 1962) - the idea is to collect data from two light cones separated by 10-20 years to look for a change in redshift of a source as a function of time.



There is a relation between the times of emission of light by the source τ_e and $\tau_e + \delta\tau_e$ and times of their observation at τ_o and $\tau_o + \delta\tau_o$:

$$\int_{\tau_e}^{\tau_o} \frac{d\tau}{a(\tau)} = \int_{\tau_e+\delta\tau_e}^{\tau_o+\delta\tau_o} \frac{d\tau}{a(\tau)} , \tag{44}$$

which for small $\delta\tau_e$ and $\delta\tau_o$ reads as $\frac{\delta\tau_e}{a(\tau_e)} = \frac{\delta\tau_o}{a(\tau_o)}$.

Redshift drift in inhomogeneous pressure models.

For small $\delta\tau_e$ and $\delta\tau_o$ we expand in Taylor series

$$\begin{aligned}(u_a k^a)_o &= (u_a k^a)(r_0, \tau_0 + \delta\tau_0) = (u_a k^a)(r_0, \tau_0) + \left[\frac{\partial(u_a k^a)}{\partial\tau}\right]_{(r_0,\tau_0)} \delta\tau_0 \\ (u_a k^a)_e &= (u_a k^a)(r_e, \tau_e + \delta\tau_e) = (u_a k^a)(r_e, \tau_e) + \left[\frac{\partial(u_a k^a)}{\partial\tau}\right]_{(r_e,\tau_e)} \delta\tau_e \ ,\end{aligned}$$

where for inhomogeneous pressure models the readshift reads as

$$1 + z = \frac{(u_a k^a)_e}{(u_a k^a)_O} = \frac{\frac{V(t_e,r_e)}{R(t_e)}}{\frac{V(t_0,r_0)}{R(t_0)}} \tag{45}$$

From the definition of the redshift drift by Sandage (1962):

$$\delta z = z_e - z_0 = \frac{(u_a k^a)(r_e, \tau_e + \delta\tau_e)}{(u_a k^a)(r_0, \tau_0 + \delta\tau_0)} - \frac{(u_a k^a)(r_e, \tau_e)}{(u_a k^a)(r_0, \tau_0)} \ , \tag{46}$$

Redshift drift in inhomogeneous pressure models.

For general SS metric we obtain

$$\frac{\partial}{\partial\tau}\left(u_a k^a\right) = -\left(\frac{1}{a}\right)^{\cdot} - \frac{1}{4}\left(\frac{k}{a}\right)^{\cdot} r^2 \, , \tag{47}$$

and

$$\frac{\delta z}{\delta\tau_0} = \frac{\left[\left(\frac{1}{a}\right)^{\cdot} - \frac{1}{4}\left(\frac{k}{a}\right)^{\cdot} r^2\right]_e}{\left[1+\frac{1}{4}kr^2\right]_e} a(\tau_e) - \frac{\left[\left(\frac{1}{a}\right)^{\cdot} + \frac{1}{4}\left(\frac{k}{a}\right)^{\cdot} r^2\right]_o}{\left[1+\frac{1}{4}kr^2\right]_o} a(\tau_0)(1+z) \tag{48}$$

For the model with $(k/a)^{\cdot} = 0$ we have

$$\frac{\delta z}{\delta\tau} = -\frac{H_0}{1+\frac{1}{4}k(\tau_0)r_0^2}\left[\frac{H_e}{H_0} - (1+z)\right] \, , \tag{49}$$

where $H_e \equiv H(\tau_e) = \dot{a}(\tau_e)/a(\tau_e)$.

Redshift drift in inhomogeneous pressure models.

Eventually we end up with the following set of formulas that combined together allows us to find the rate of change of redshift $\frac{\delta z}{\delta \tau}$ (**a redshift drift**) of any source at redshift z in the considered class of Stephani model defined by the relation $k(\tau) = \beta a(\tau)$.

$$\frac{\delta z}{\delta \tau} = -\frac{H_0}{1+\frac{1}{4}H_0^2(\Omega_0 - 1)\tilde{r}_0^2}\left(\sqrt{\Omega_0 \tilde{a}^{-3(w+1)} + (1-\Omega_0)\tilde{a}^{-1}} - 1 - z\right) \quad (50)$$

$$\tilde{a}^{-1} = \frac{1+\frac{H_0^2}{4}(\Omega_0 - 1)\tilde{r}_0^2}{1+\frac{H_0^2}{4}(\Omega_0 - 1)\tilde{a}\tilde{r}^2}(1+z), \quad (51)$$

$$\frac{d\tilde{r}}{d\tau} = \pm \tilde{a}^{-1}\left(1 - \frac{\tilde{r}_0^2}{\tilde{r}^2} sin^2\phi\right)^{1/2}. \quad (52)$$

where the last equation describes the propagation of the null geodesic.

Redshift drift in inhomogeneous pressure models.

In the limit where $\Omega_0 = 1 \Rightarrow \Omega_{inh} = 0$ and $w = 0$, i.e. **a flat FRW model filled with dust (CDM)** the formula (50) reduces to

$$\frac{\delta z}{\delta \tau} = -H_0 [(1+z)^{3/2} - (1+z)] \; , \tag{53}$$

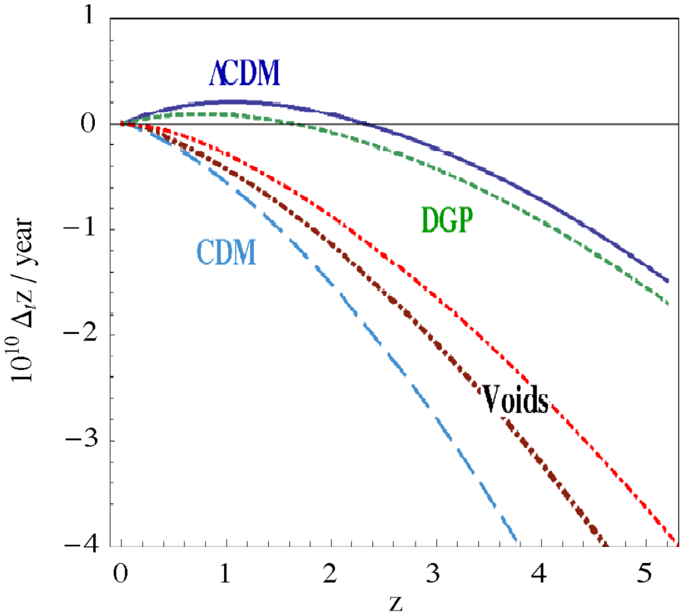
which coincides with the formulas obtained in earlier papers investigating the problem (Sandage 1962, Loeb 1998).

On the other hand, for **pressure-inhomogeneity-dominated universe** $\Omega_0 \to 0 \Rightarrow \Omega_{inh} \to 1$, and we have a simple result

$$\frac{\delta z}{\delta t} = H_0 \frac{z}{2} \; , \tag{54}$$

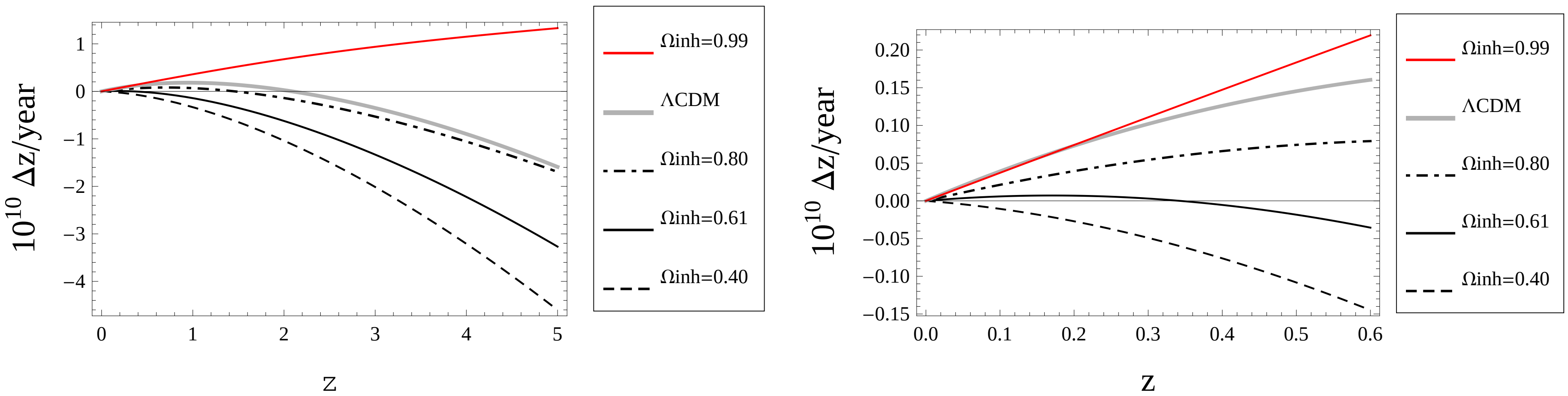
which means that the drift **grows linearly** with redshift.

Redshift drift in cosmological models.



- Quercellini et. al (2012) found the redshift drift for: ΛCDM, DGP model, Cold Dark Matter (CMD) model, 3 different void models (LTB).
- ΛCDM, DGP - the drift is **positive at small redshift**, but becomes negative for $z \gtrsim 2$
- Giant void (LTB) model mimicking dark energy - the drift is **always negative**.

Redshift drift in inhomogeneous pressure models.



- The redshift drift for the sperically symmetric inhomogeneous pressure Stephani model with $r_0 = 0$, $w = 0$.
- Ω_{inh} (parameter of inhomogeneity) **small** - mimics LTB and CDM models
- Ω_{inh} **larger** - the drift alike in ΛCDM models (first positive, then negative), e.g. for $\Omega_{inh} = 0.61$ drift is positive for $z \in (0, 0.34)$.
- Ω_{inh} **very large** - drift positive ($\Omega_{inh} = 0.99$ up to $z = 17$; $\Omega_{inh} = 1$ (inhomogeneity-domination) $z > 0$).

Redshift drift - future observations.

- LTB (void) inhomogeneity (due to the energy density) is different from the Stephani inhomogeneity (due to the pressure) which shows in the fact that the drift is always negative for an LTB model and always positive for an inhomogeneity-dominated Stephani model.
- One is able to differentiate between the drift in ΛCDM models, in LTB models, and in Stephani models - this can be done in future experiments.
- At larger $z > 1.7$ redshifts by giant telescopes: European Extremely Large Telescope (E-ELT) with spectrograph CODEX (COsmic Dynamics EXperiment); Thirty Meter Telescope (TMT); Giant Magellan Telescope (GMT).
- At smaller (even $z \sim 0.2$) redshifts by gravitational wave interferometers DECIGO/BBO (DECi-hertz Interferometer Gravitational Wave Observatory/Big Bang Observer). **This could clearly reject LTB models if the drift measured was positive!**

5. Other tests: luminosity distance, baryon acoustic oscillations (BAO), shift parameter. Centrally placed observer.

The luminostity distance for a central observer $r_0 = 0$ is (same as Friedmann)

$$d_L = (1+z)a_0 r \ , \tag{55}$$

and the distance modulus is

$$\mu(z) = 5 \log_{10} d_L(z) + 25. \tag{56}$$

From the null geodesic equations we have

$$r = c\int_a^{a_0} \frac{da}{\sqrt{c^2A^2a^{1-3w} - \beta c^2 a^3}} = r = \frac{c}{H_0 a_0}\int_{a/a_0}^{1} \frac{dx}{\sqrt{\Omega_0 x^{1-3w} + (1-\Omega_0)x^3}} \ , \tag{57}$$

where $x \equiv a/a_0$. Using the definition of redshift (45) one can rewrite (57) as

$$z(x) = \frac{1}{x} - 1 + \frac{\Omega_0 - 1}{4}\left[\int_{a/a_0}^{1} \frac{dx}{\sqrt{\Omega_0 x^{1-3w} + (1-\Omega_0)x^3}}\right]^2 \ , \tag{58}$$

Other tests - luminosity distance, apparent magnitude

and so the luminosity distance (55) reads as

$$d_L(x) = \frac{c(1+z)}{H_0}\sqrt{\frac{4[z(x)+1-1/x]}{\Omega_0 - 1}} \quad . \tag{59}$$

The series expansion redshift-magnitude relation for the Model I was already obtained in Dąbrowski & Hendry (1998) as follows

$$m = M + 25 + 5\log_{10}\left[cz\left(\frac{a\tau_0^2+\tau_0}{2a\tau_0+1}\right)\right] + 1.086\left[1 + 4a\frac{\left(a\tau_0^2+\tau_0\right)}{\left(2a\tau+1\right)^2}\right] z + O(z^2). \tag{60}$$

This relation has no difference with the FRW relation (with rescaled H_0 and q_0)

$$m_{bol} = M - 5\log_{10}\tilde{H}_0 + 5\log_{10}cz + 1.086\,(1-\tilde{q}_0)\,z + O(z^2), \tag{61}$$

$$\tilde{H}_0 = \frac{2a\tau_0+1}{a\tau_0^2+\tau_0}, \qquad \tilde{q}_0 = -4a\frac{a\tau_0^2+\tau_0}{(2a\tau_0+1)^2} \tag{62}$$

CMB shift parameter.

The shift parameter is defined as:

$$\mathcal{R} = \frac{l_1'^{TT}}{l_1^{TT}} \quad , \tag{63}$$

where l_1^{TT} – the temperature perturbation CMB spectrum multipole of the first acoustic peak in inh. pressure model
$l_1'^{TT}$ – the multipole of a reference flat standard Cold Dark Matter model. The multipole number is related to an angular scale of the sound horizon r_s at decoupling by

$$\theta_1 = \frac{r_s}{d_A} \propto \frac{1}{l_1} \; . \tag{64}$$

For our Stephani model the angular diameter distance is given by

$$d_A = \frac{a_{\rm dec}}{V(t_{dec}, r_{dec})} r_{\rm dec} \tag{65}$$

with $r_{\rm dec}$ given by (57) taken at decoupling.

CMB shift parameter.

Using the above, we may write that for our Stephani models the shift parameter is

$$\mathcal{R} = \frac{2cV(t_{dec}, r_{dec})}{H_0\sqrt{\Omega_0} r_{\rm dec}} \,. \tag{66}$$

Finally, the rescaled shift parameter is

$$\bar{\mathcal{R}} \quad = \quad \frac{H_0\sqrt{\Omega_0} r_{\rm dec}}{cV(t_{dec}, r_{dec})} \,. \tag{67}$$

The WMAP data gives $\bar{\mathcal{R}} = 1.70 \pm 0.03$ (Wang, Mukherjee 2006).

Baryon acoustic oscillations.

One calculates the distortion of a spherical object in the sky without knowing its true size by measuring its transverse extent using the angular diameter distance, r

$$r = \frac{l}{\Delta\theta} \; , \tag{68}$$

where l and $\Delta\theta$ are the linear and angular size of an object, and its line-of-sight extent, Δr, using the redshift distance

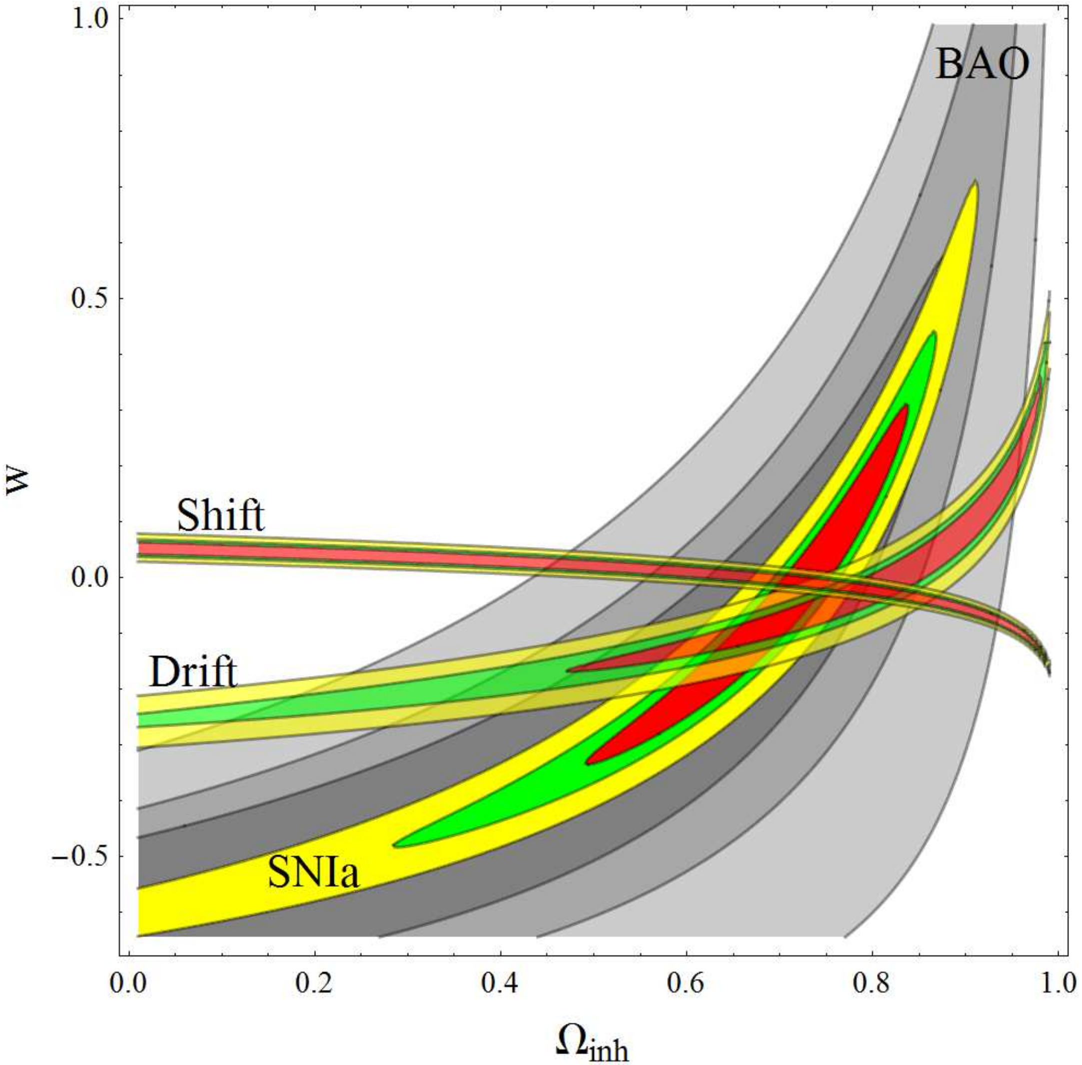
$$\Delta r = \frac{c\Delta t}{a(t)} \tag{69}$$

(see e.g. Nesseris (2006)). As a result one can define the volume distance, D_V, as

$$D_V^3 = r^2 \Delta r \; . \tag{70}$$

Eisenstein et al. (2005) gave $D_V(\Delta z = z_{BAO} = 0.35) = 1370 \pm 64$ Mpc (an acoustic peak for 46748 luminous red galaxies (LRG) selected from the SDSS (Sloan Digital Sky Survey).

Inhomogeneous pressure - combined tests (SNIa, RD, BAO, shift parameter)



Confidence intervals (contours of 68%, 95% and 99% credible regions).

Inhomogeneous pressure - combined tests: results and improvements

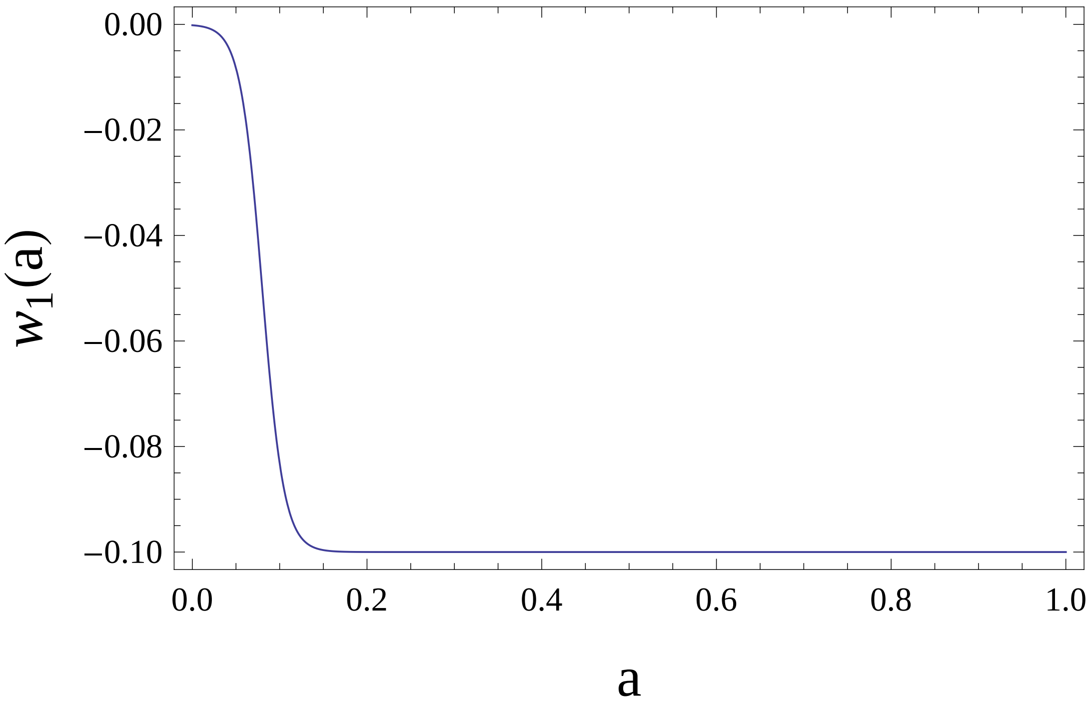
- Stephani model fits well the data for the SNIa, redshift drift, and BAO (contours overlap at 1σ CL).
- It cannot fit the existing observational data and recover at the same time the redshift drift features of the ΛCDM model (at least within 1σ CL).
- Way out: replace constant barotropic index w **by** $w_1(a)$**.**
- One does not want to change the contours obtained for SNIa, BAO, and redshift drift so one postulates the function $w(a)$ to be constant on the redshift interval encompassing all the redshifts from now ($z = 0$) up to the most redshifted source of the redshift drift at $z = 5$.
- One assumes that $w(a)$ suddenly changes somewhere between $z = 5$ and z_{dec}, and then remains constant.
- This allows to lower the contours obtained for the shift parameter so that in the resulting plot all the contours will be overlapping.

Inhomogeneous pressure models - $w_1(a)$ parametrization

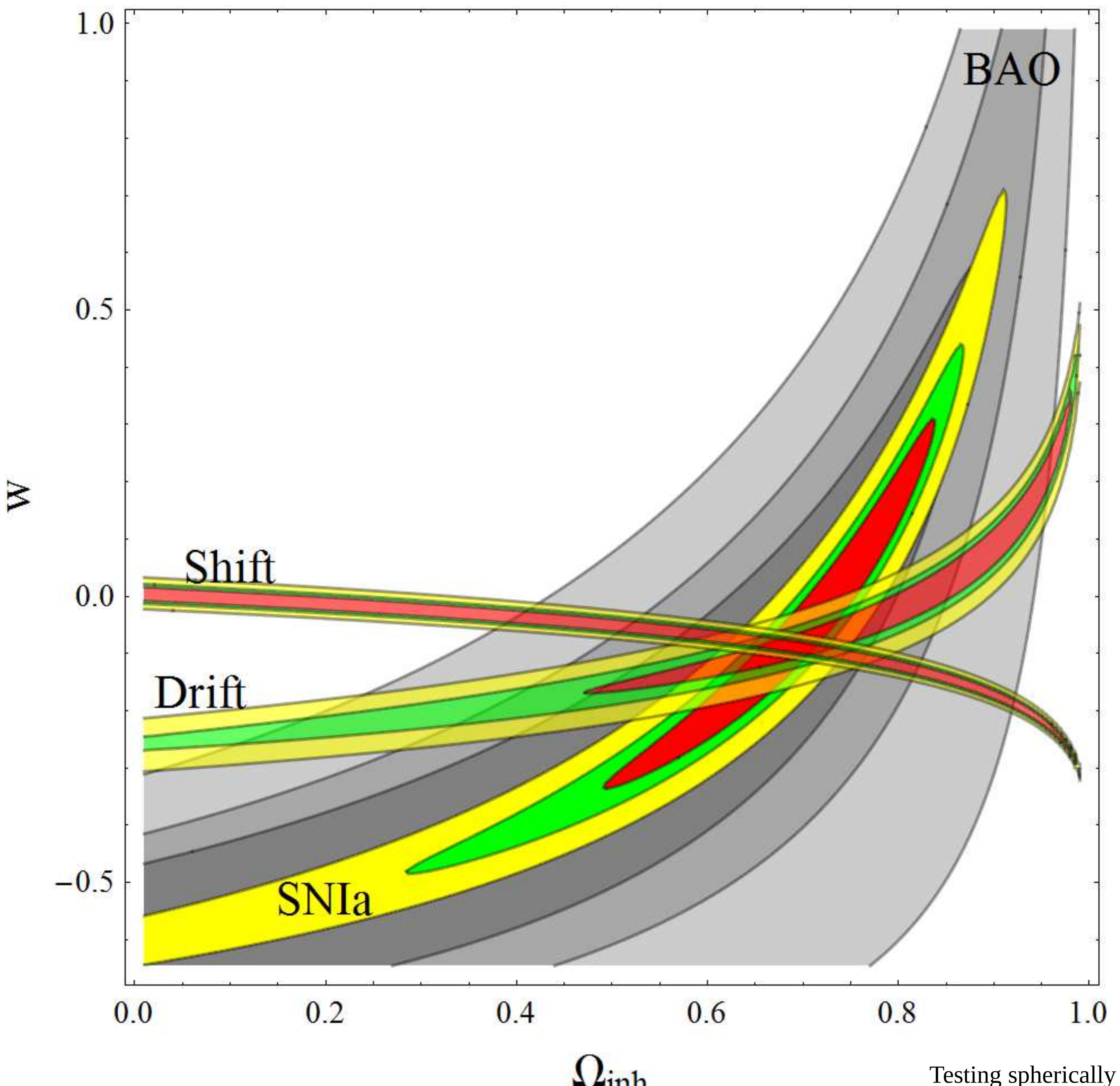
An example of the function $w_1(a)$ which can fit the data is:

$$w_1(a) = w + \frac{w_0}{2}\left(1 + \tanh[\lambda(a_{tr} - a)]\right) \ . \qquad (71)$$

where w, w_0, λ, and a_{tr} are constants. Here: $\lambda = 40$, $a_{tr} = 0.08$, $w_0 = 0.1$, $w = -0.1$ and $\Omega_{inh} = 0.68$, $z_{tr} \sim 10.66$.



Inhomogeneous pressure models - combined tests for $w_1(a)$



Inhomogeneous pressure models - combined tests for $w_1(a)$

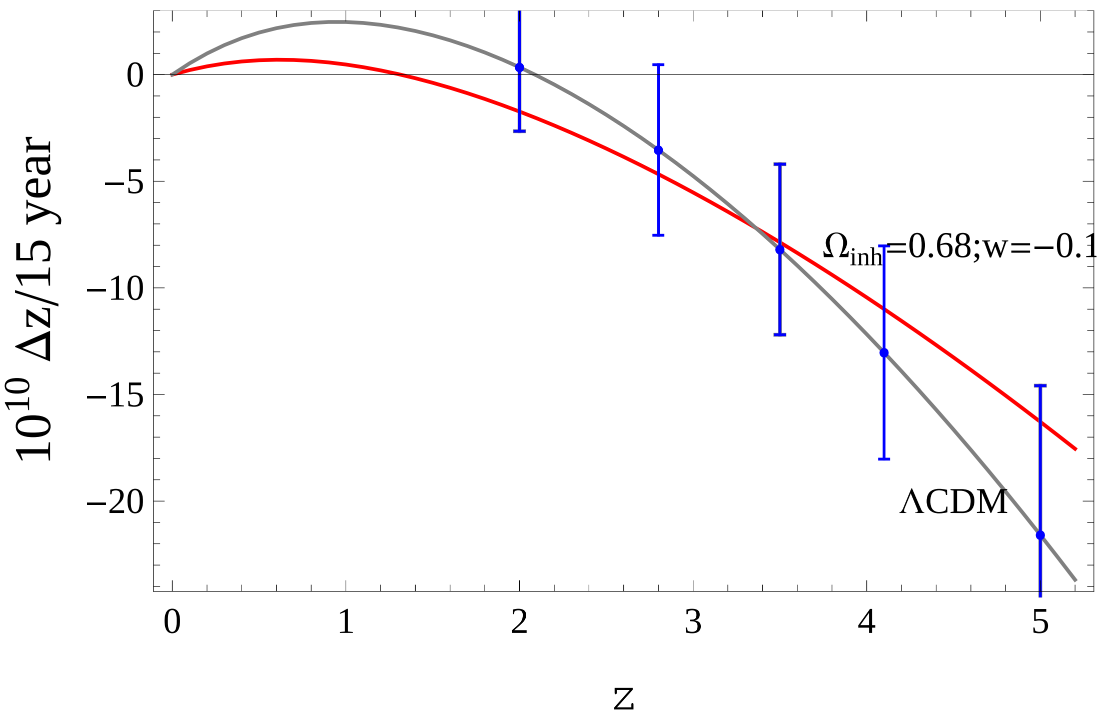
As expected the Stephani model with the scale factor dependent barotropic index $w_1(a)$ (71) and $\lambda = 40$, $a_{tr} = 0.08$ and $w_0 = 0.1$ agrees with the current observational data for the SNIa, BAO and the shift parameter and at the same time recovers most features of the redshift drift relation in the ΛCDM model.
CODEX Monte Carlo simulated error on the measured spectroscopic velocity shift is:

$$\sigma_{\Delta v} = 1.35 \frac{2370}{S/N} \sqrt{\frac{30}{N_{QSO}}} \left(\frac{5}{1 + z_{QSO}} \right)^{1.7} cm/s \ , \qquad (72)$$

where S/N is signal to noise ratio, N_{QSO} number of observed quasars.

Inhomogeneous pressure - combined tests

For the redshift drift we use the “fake” data set presented in Quercellini et al. (2012) (see the blue error bars). This data set is assumed to be centered on the ΛCDM redshift drift curve and to have normally distributed errors.



6. Off-center observers and supernovae.

The luminosity distance is given by

$$d_L = \frac{a_0(1+z)\hat{r}'}{1+\frac{\beta}{4}a_0 r_0^2} \; , \tag{73}$$

with an off-center observer placed at r_0, θ_0, ϕ_0 as meant in the coordinate system $\{t, r, \theta, \varphi\}$ of the Stephani metric. More precisely we have

$$d_L = \frac{(1+z)}{1-\frac{a_0 H_0^2 \Omega_{inh}}{4} r_0^2} \, \hat{r}'(\Omega_{inh}, w, r_0, \theta_0, \varphi_0, H_0, \hat{\theta}', \hat{\varphi}', z) \; , \tag{74}$$

where

$$\hat{r}' = \hat{r}'(a) = \frac{1}{H_0} \int_{a_e}^{1} \frac{dx}{\sqrt{(1-\Omega_{inh})x^{1-3w} + \Omega_{inh} x^3}} \; , \tag{75}$$

and a_e is the value of the scale factor at the moment of an emission of the light ray.

Off-center observers

For the redshift one takes

$$1 + z = \frac{a_0 (4 - a_e H_0^2 \Omega_{inh} r_e^2)}{a_e (4 - a_0 H_0^2 \Omega_{inh} r_0^2)} \, , \tag{76}$$

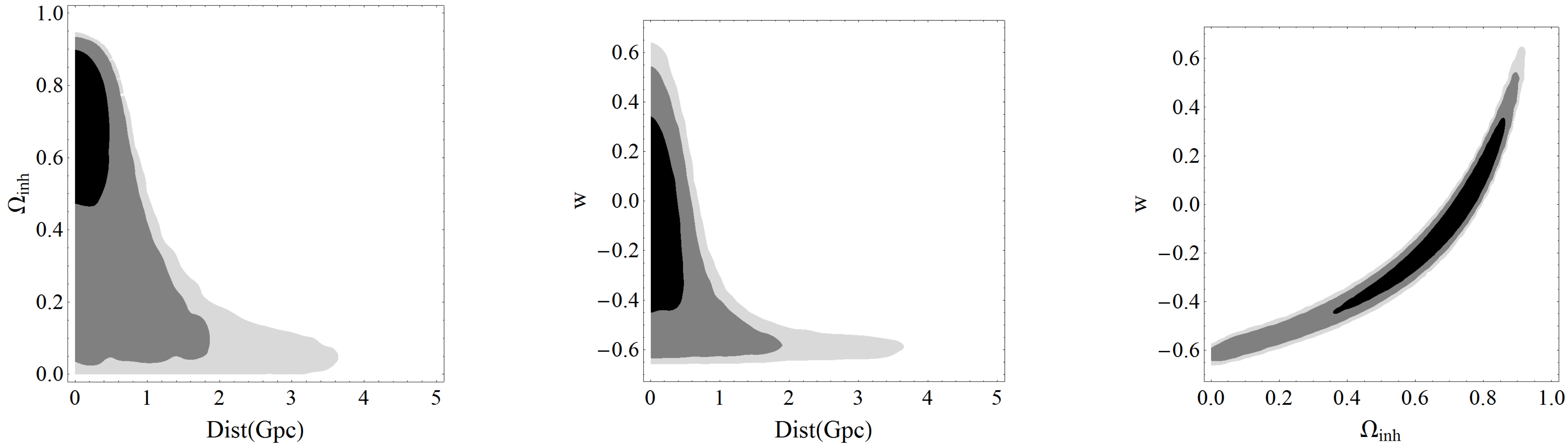
where

$$\begin{aligned} r_e^2 &= (r_0 \sin\theta_0 \cos\varphi_0 + \hat{r}'(a) \sin\hat{\theta}' \cos\hat{\varphi}')^2 \\ &+ (r_0 \sin\theta_0 \sin\varphi_0 + \hat{r}'(a) \sin\hat{\theta}' \sin\hat{\varphi}')^2 \\ &+ (r_0 \cos\theta_0 + \hat{r}'(a) \cos\hat{\theta}' \sin\hat{\varphi}')^2 \end{aligned} \tag{77}$$

and $\hat{\theta}'$ and $\hat{\varphi}'$ are the coordinates of a supernova as seen by an off-center observer in the sky.

We applied Union2 557 supernovae data of Amanullah et al. (2010, ApJ, 716, 712) - we note the courtesy of M. Kowalski and U. Feindt to consult the sample.

Off-center observers



Best-fit values: inhomogeneity density $\Omega_{inh} \sim 0.74$, center of symmetry equation of state barotropic index $w \sim 0.031$, off-center observer position $Dist = 270$ Mpc $(\chi^2 = 339)$.

7. Conclusions

- Observations from one point in the Universe **suggest its isotropy**, but not necessarily homogeneity. This gives **motivation** for studying spherically symmetric models of the Universe.
- Two specific models have been proposed: the **Lemaître-Tolman-Bondi** model (inhomogeneous density) and the **Stephani** model (inhomogeneous pressure).
- These models have been preliminary **checked against astronomical data** which shows that the **inhomogeneities may drive acceleration**.
- Inhomogeneous pressure models have another advantage - they can even model **a total spacetime inhomogeneity**.
- There is an open question whether we **really live in** a homogeneous and isotropic (FRW) universe or at least in an isotropic (spherically symmetric) **void or an interior of an inhomogeneous pressure "exotic star"**.

conclusions contd.

- In the class of Stephani models considered (with a centrally placed observer) there is a subset of observationally viable models which **show qualitatively different behavior** of redshift drift than the LTB void models and ΛCDM models whic can be tested by veray large telescopes and GW detectors.
- Stephani model fits well the data for the SNIa, redshift drift, and BAO though it does not recover the redshift drift features of the ΛCDM model.
- However, it can fit all the data SNIa, redshift drift, shift parameter, and BAO provided a specific parametrization for $w_1(a)$ is applied.
- Comparison with the 557 Union2 supernovae data restricts the position of non-centrally placed observers to be not more than 2-4 Gpc away from the center with best-fit value of Dist = 270 Mpc.