The properties and classification of exotic singularities in cosmology

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References

- MPD, PRD **71**, 103505 (2005) (gr-qc/0410033)
- MPD, PLB **625**, 184 (2005) (gr-qc/0505069)
- A. Balcerzak and MPD, PRD **73**, 101301(R) (2006) (hep-th/0604034)
- MPD, T. Denkiewicz, M.A. Hendry, PRD **75**, 123524 (2007)
- MPD, T. Denkiewicz, PRD **79**, 063521 (2009)
- MPD, T. Denkiewicz, 0910.0023 AIP Conference Proceedings **1241**, 561 (2010).
- MPD, Ann. Phys. (Leipzig) **19**, 299 (2010)
- MPD, T. Denkiewicz, M.A. Hendry, H. Ghodsi, work in progress
- MPD, work in progress

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1. Standard Big-Bang/Crunch (type 0) versus exotic singularities.

Standard Einstein-Friedmann equations are two equations for three unknown functions of time $a(t), p(t), \varrho(t)$

$$\varrho = 3\left(\frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\right) , \tag{1}$$

$$p = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\right) . \tag{2}$$

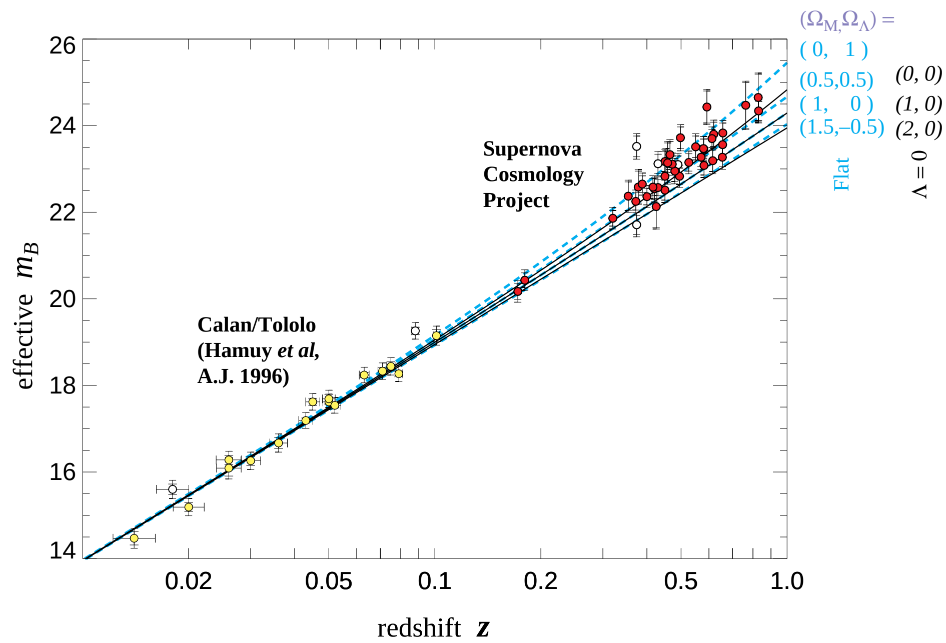
plus an equation of state, e.g., of a barotropic type ($w =$ const. ≥ -1):

$$p(t) = w\varrho(t) \qquad \rightarrow a(t) \propto t^2 3(w+1) . \tag{3}$$

Until very recently (including first supernovae results) most of cosmologists studied only simplest - say "standard" solutions - each of them starts with Big-Bang singularity in which $a \rightarrow 0$, $\varrho, p \rightarrow \infty$
– one of them (of $K = +1$) terminates at the second singularity (Big-Crunch) where $a \rightarrow 0$, $\varrho, p \rightarrow \infty$
– the other two ($K = 0, -1$) continue to an asymptotic emptiness $\varrho, p \rightarrow 0$ for $a \rightarrow \infty$.

BB and BC exhibit geodesic incompleteness and curvature blow-up.

First supernovae observations ...



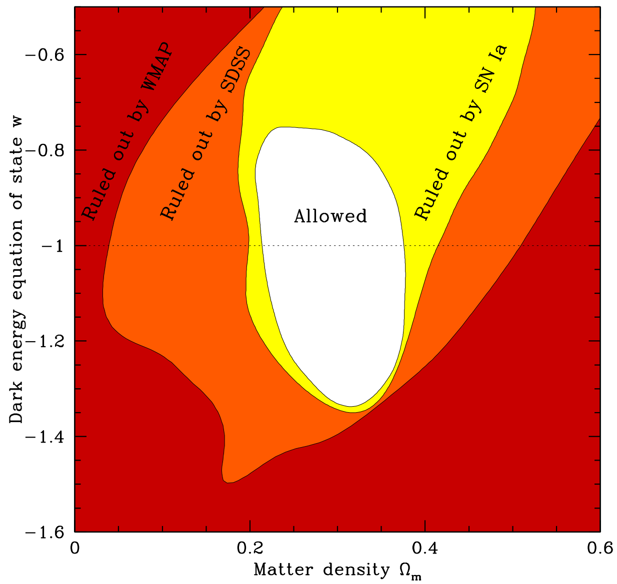
... gave evidence for the strong energy condition

$$\varrho + 3p \geq 0, \qquad \varrho + p \geq 0\,. \tag{4}$$

violation, but **the paradigm of the "standard" Big-Bang/Crunch singularities remained untouched.**

2. Big-Rip (type I) as an exotic singularity.

However, WMAP + SDSS + Supernovae **combined bound on the dark energy barotropic index** w (Tegmark et al. (2004)):



- **showed that there was no sharp cut-off of the data at $p = -\varrho$!!!** so that the
- **dark energy with $p < -\varrho$ (phantom) can be admitted!**

More recent data:

- Knop et al. 2003 (from SNe + CMB + 2dFGRS combined) – $w = -1.05^{+0.15}_{-0.20}$ (statistical) ± 0.09 (systematic)
- Riess et al. 2004 ($w < -1$)
- Seljak et al. astro-ph/0604335 – $w = -1.04 \pm 0.06$
- though more recently Kowalski et al. (arXiv:0804.4142) analyzed 307 supernovae (Sne + BAO + CMB) – $w = -1.001^{+0.059}_{-0.063}$ (statistical) $^{+0.063}_{-0.066}$ (systematic)

gave some evidence for possible cosmic "no-hair" theorem violation - **even a small fraction of phantom dark energy may dominate the evolution**

N(ull) E(nergy) C(ondition) $\varrho + p \geq 0$,

W(eak) E(nergy) C(ondition) $\varrho + p \geq 0, \rho \geq 0$,

D(ominant) E(nergy) C(ondition) $\mid p \mid \leq \varrho, \varrho \geq 0$ are violated!!!

Big-Rip (type I) as an exotic (neither BB nor BC) singularity.

Since for phantom $w < -1$, then for convenience we may take

$$\mid w+1 \mid = -(w+1) > 0 \,, \tag{5}$$

so $a(t) = t^{-2/3|w+1|}$ and the conservation law for phantom gives

$$\varrho \propto a^{3|w+1|} \,. \tag{6}$$

- Conclusion: the bigger the universe grows, the denser it is, and **it becomes dominated by phantom (which overcomes Λ-term)** – an exotic future singularity appears – Big-Rip $\varrho, p \to \infty$ for $a \to \infty$
- Curvature invariants R^2, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ diverge at Big-Rip
- Only for $-5/3 < w < -1$ the null geodesics are geodesically complete; for other values of w, including all timelike geodesics, there is a geodesic incompleteness (Lazkoz et al. gr-qc/0607073, PRD 2006) - the singularity is reached in a finite proper time.

3. Sudden Future Singularity (type II) as an exotic singularity.

Observational support for a Big-Rip gave a push to studies some other exotic types of singularities as possible sources of dark energy.

Barrow (2004) proposed a Sudden Future Singularity (SFS) (or type II - Nojiri, Odintsov, Tsujikawa 2005) which:

– manifests as a singularity of pressure (or $\ddot{a}$) only

– leads to the dominant energy condition violation only and it emerges due to

a drop of the assumption about the imposition of an equation of state

$$p \neq p(\varrho), \qquad \text{no analytic form of this relation is given} \tag{7}$$

Only the form of the scale factor is given in the field equations:

$$a(t) = a_s \left[\delta + (1-\delta)\, y^m - \delta\, (1-y)^n\right] \,, \qquad y \equiv \frac{t}{t_s} \tag{8}$$

where $a_s \equiv a(t_s) = \text{const.}$ and $\delta, m, n = \text{const.}$

Apart from a Big-Bang at $t = 0$ there is a new type of singularity at $t = t_s$.

$$\dot{a} = a_s \left[\frac{m}{t_s} (1-\delta)\, y^{m-1} + \delta \frac{n}{t_s} (1-y)^{n-1} \right] , \tag{9}$$

$$\ddot{a} = \frac{a_s}{t_s^2} \left[m(m-1)(1-\delta)\, y^{m-2} - \delta n (n-1)(1-y)^{n-2} \right] . \tag{10}$$

Provided

$$1 < n < 2, \tag{11}$$

and using Einstein equations we get the following properties:

$$\begin{array}{cccc} a = \text{const.}, & \dot{a} = \text{const.} & \varrho = \text{const.} & \\ \ddot{a} \to -\infty & p \to \infty & \text{for} & t \to t_s \end{array} \tag{12}$$

Friedmann limit is easily obtained by taking the “nonstandardicity” parameter $\delta \to 0$.

Generalized Sudden Future singularities.

Sudden future singularities may be generalized to GSFS if we take a general scale factor time derivative of an order r:

$$\begin{aligned} a^{(r)} &= a_s \left[\frac{m(m-1)...(m-r+1)}{t_s^r} (1-\delta)\, y^{m-r} \right. \\ &+ \left. (-1)^{r-1} \delta \frac{n(n-1)...(n-r+1)}{t_s^r} (1-y)^{n-r} \right] , \end{aligned} \qquad (13)$$

and choose (Barrow 2004, Lake 2004) $r-1 < n < r$. Then for any integer r we have a singularity in the scale factor derivative $a^{(r)}$, and consequently in the appropriate pressure derivative $p^{(r-2)}$.

None of the energy conditions are violated for $r \geq 3$!!!

4. Finite Scale Factor (type III), Big Separation (type IV) and w-singularities (type V).

Type III singularities which we will call Finite Scale Factor - FSF singularities are characterized by the following conditions:

$$a = a_s = \text{const.}, \varrho, \dot{a}_s \to \infty, |p|, \ddot{a}_s \to \infty$$

The simplest way to get them is to apply the scale factor as given previously for SFS, i.e.,

$$a(t) = a_s \left[\delta + (1-\delta)\, y^m - \delta\, (1-y)^n\right] \, , \qquad y \equiv \frac{t}{t_s} \tag{14}$$

where $a_f \equiv a(t_f) = \text{const.}$ and $\delta, A, m, n = \text{const.}$, but with the range of parameter n changed from $1 < n < 2$ onto

$$0 < n < 1$$

Big Separation - BS (type IV)

Type IV singularity is when:

$$a = a_s = \text{const.},\ \varrho \to 0,\ p \to 0,\ \dddot{a},\ \ddot{H} \to \infty \text{ etc.}$$

and so it is similar to Generalized Sudden Future singularity with only one exception: it also gives the divergence of the barotropic index in the barotropic equation of state

$$p(t) = w(t)\varrho(t)$$

i.e.,

$$w(t) \to \infty$$

Barotropic index $w-$singularity

Another exotic is a $w-$singularity only (without the divergence of the higher-derivatives of the scale factor). (Strangely, it really appears in physical theories such as $f(R)$ gravity (Starobinsky '80), scalar field gravity (Setare, Saridakis '09, and brane gravity Sahni, Shtanov '05)). We choose

$$a(t) = A + B\left(\frac{t}{t_s}\right)^{\frac{2}{3\gamma}} + C\left(D - \frac{t}{t_s}\right)^{n} , \tag{15}$$

where A, B, C, D, γ, n, and t_s are constants and impose the conditions:

$$a(0) = 0,\ a(t_s) = const. \equiv a_s,\ \dot{a}(t_s) = 0,\ \ddot{a}(t_s) = 0 , \tag{16}$$

which finally leads to the following form of the scale factor:

$$
\begin{aligned}
a(t) &= \frac{a_s}{1 - \frac{3\gamma}{2}\left(\frac{n-1}{n-\frac{2}{3\gamma}}\right)^{n-1}} \\
&+ \frac{1-\frac{2}{3\gamma}}{n-\frac{2}{3\gamma}} \frac{n a_s}{1-\frac{2}{3\gamma}\left(\frac{n-\frac{2}{3\gamma}}{n-1}\right)^{n-1}} \left(\frac{t}{t_s}\right)^{\frac{2}{3\gamma}} \\
&+ \frac{a_s}{\frac{3\gamma}{2}\left(\frac{n-1}{n-\frac{2}{3\gamma}}\right)^{n-1} - 1} \left(1 - \frac{1-\frac{2}{3\gamma}}{n-\frac{2}{3\gamma}} \frac{t}{t_s}\right)^{n} ,
\end{aligned} \tag{17}
$$

with the admissible values of the parameters: $\gamma > 0$ and $n \neq 1$.

$w-$duality

We have a blow-up of the effective barotropic index, i.e.,

$$w(t_s) = \frac{c^2}{3}\left[2q(t_s) - 1\right] \to \infty \; , \tag{18}$$

accompanied by

$$p(t_s) \to 0; \quad \varrho(t_s) \to 0 \; . \tag{19}$$

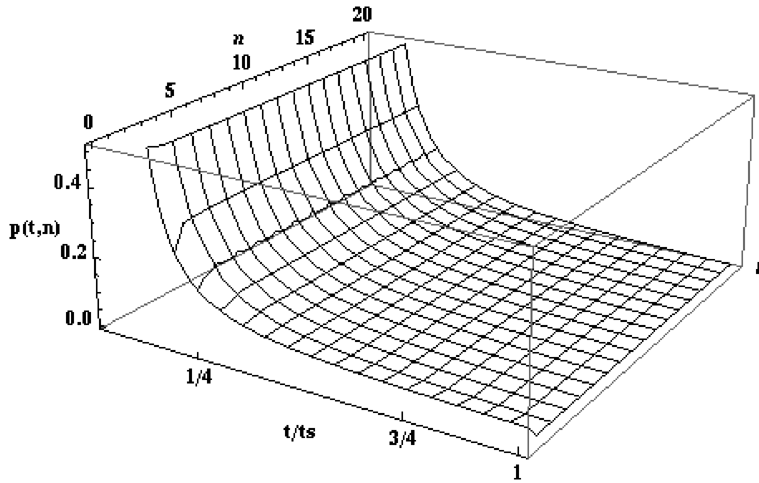
There is an amazing duality between the Big-Bang and the w-singularity in the form

$$p_{BB} \leftrightarrow \frac{1}{p_w}, \quad \varrho_{BB} \leftrightarrow \frac{1}{\varrho_w}, \quad w_{BB} \leftrightarrow \frac{1}{w_w} \quad . \tag{20}$$

In other words:

$$p_{BB} \to \infty; \; \varrho_{BB} \to \infty; \; w_{BB} \to 0; \; a_{BB} \to 0$$
$$p_w \to 0; \; \varrho_w \to 0; \; w_w \to \infty; \; a_w \to a_s = \text{const.}$$

$w-$duality



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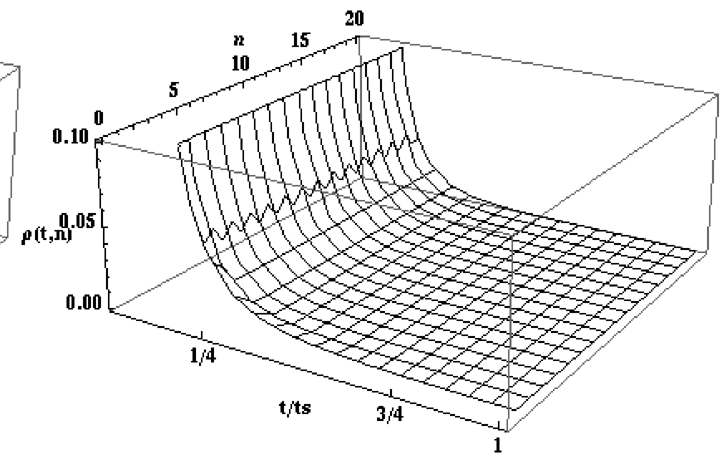

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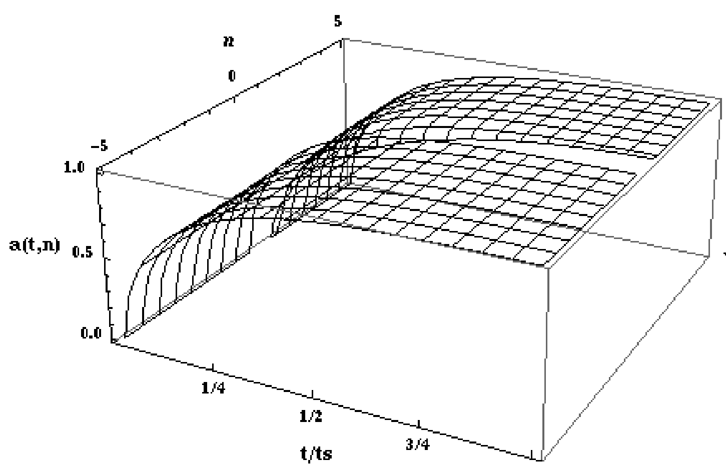

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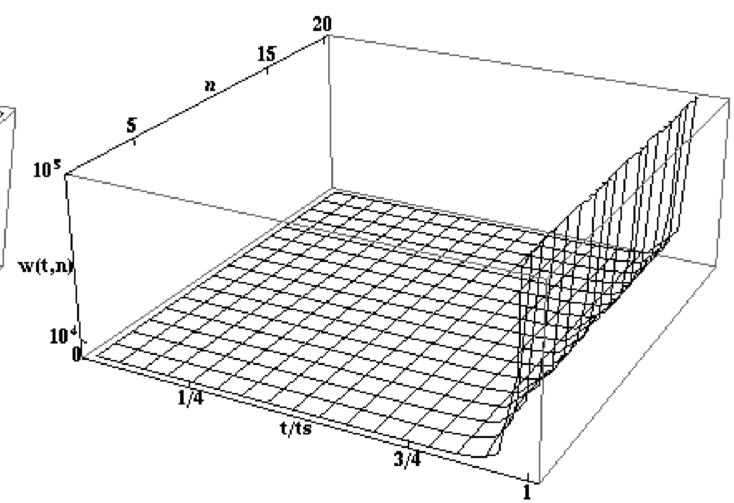

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t/ts
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Strength of exotic singularities.

SFSs are determined by a blow-up of the Riemann tensor and its derivatives. Geodesics do not feel SFSs at all, since geodesic equations are not singular for $a_s = a(t_s) =$ const. (Fernandez-Jambrina, Lazkoz PRD 74, 064030 (2006) ((gr-qc/0607073))

$$\left(\frac{dt}{d\tau}\right)^2 = A + \frac{P^2 + KL^2}{a^2(t)} , \tag{21}$$

$$\frac{dr}{d\tau} = \frac{P_1 cos\phi + P_2 \sin\phi}{a^2(t)} \sqrt{1 - Kr^2} , \tag{22}$$

$$\frac{d\phi}{d\tau} = \frac{L}{a^2(t) r^2} . \tag{23}$$

Geodesic deviation equation

$$\frac{D^2 n^\alpha}{d\lambda^2} + R^\alpha_{\beta\gamma\delta} u^\beta n^\gamma u^\delta = 0 , \tag{24}$$

feels SFS since at $t = t_s$ we have the Riemann tensor $R^\alpha_{\beta\gamma\delta} \longrightarrow \infty$.

No geodesic incompleteness.

- No geodesic incompleteness ($a =$ const. and r.h.s. of geodesic eqs. do not diverge) $\Rightarrow$ SFS are not the final state of the universe
- Point particles do not even see SFSs while extended objects may suffer instantaneous infinite tidal forces but still are not crushed - strings can pass through SFSs in the sense that their invariant size remains finite (MPD, Balcerzak 2006).
- Tipler's (Phys. Lett. A64, 8 (1977)) definition (of a weak singularity):
  $\int_0^{\tau} d\tau' \int_0^{\tau'} d\tau'' R_{ab} u^a u^b$
  does not diverge on the approach to a singularity at $\tau = \tau_s$
- Królak's (CQG 3, 267 (1988)) definition (of a weak singularity):
  $\int_0^{\tau} d\tau' R_{ab} u^a u^b$
  does not diverge on the approach to a singularity at $\tau = \tau_s$
- Conclusion: an SFSs is different from Big-Bang or Big-Rip.

Classification of exotic singularities.

- Type 0 - Big-Bang $a \to 0$, $p \to \infty$, $\varrho \to \infty$
- Type I - Big-Rip $a \to \infty$, $p \to \infty$, $\varrho \to \infty$
- Type II - Sudden Future (includes Big Boost and Big-Brake) $a =$ const., $\varrho =$ const., $p \to \infty$
- Type IIg - Generalized Sudden Future a= const., $\varrho =$ const., $p =$const., $\dddot{a} \to \infty$ etc., $w < \infty$
- Type III - Finite Scale Factor (also Big-Freeze) $a = a_s$ = const., $\varrho \to \infty$, $p \to \infty$
- Type IV - Big Separation: a= const., $p = \varrho = 0$, $w \to \infty$, $\dddot{a} \to \infty$ etc. (and generalizations $p = \varrho =$const. Yurov 2010)
- Type V - w-singularity a= const., $p = \varrho = 0$, $w \to \infty$ (and generalizations $p =$const. Yurov 2010)

Classification of exotic singularities - strength.

Fernandez-Jambrina (PRD 82, 124004 (2010)) used Puiseux series expansion

$$a(t) = c_0 + (t_s - t)^{\eta_0} + c_1 (t_s - t)^{\eta_1} + c_2 (t_s - t)^{\eta_2} + \ldots \qquad \eta_0 < \eta_1 < \ldots \quad c_0 > 0 \tag{25}$$

to show the strengths of these singularities as follows (T - Tipler's definition; K - Królak's definition)

- Type 0 (BB, BC): T, K - strong
- Type I (BR): T, K - strong
- Type II (SFS): T, K - weak
- Type IIg (GSFS): T, K - weak
- Type III (FSF): T - weak, K - strong
- Type IV (BS): T, K - weak
- Type V (w-sing.): T, K - weak

5. Exotic singularities against astronomical data.

- There is always some fundamental physical theory (scalar field, higher-oder, string, brane, LQC) which can be related to the models with exotic singularities.
- In other words, the evidence for an exotic singularity may be attached to some form of matter which gives current acceleration of the universe and makes a candidate for the dark energy.
- We can check which of these exotic singularity universes can really serve that by checking them against data which favors accelerated universe.
- The best studied models are of course phantom models which still are within the range of observational limit - cf. Section II.
- However, it can be shown that some other models (in particular SFS models) can play a good candidate for modeling the universe.

Test of SFS (type II) models against supernovae.

We have

$$m(z) = M - 5\log_{10} H_0 + 25 + 5\log_{10}[r_1 a(t_0)(1+z)], \tag{26}$$

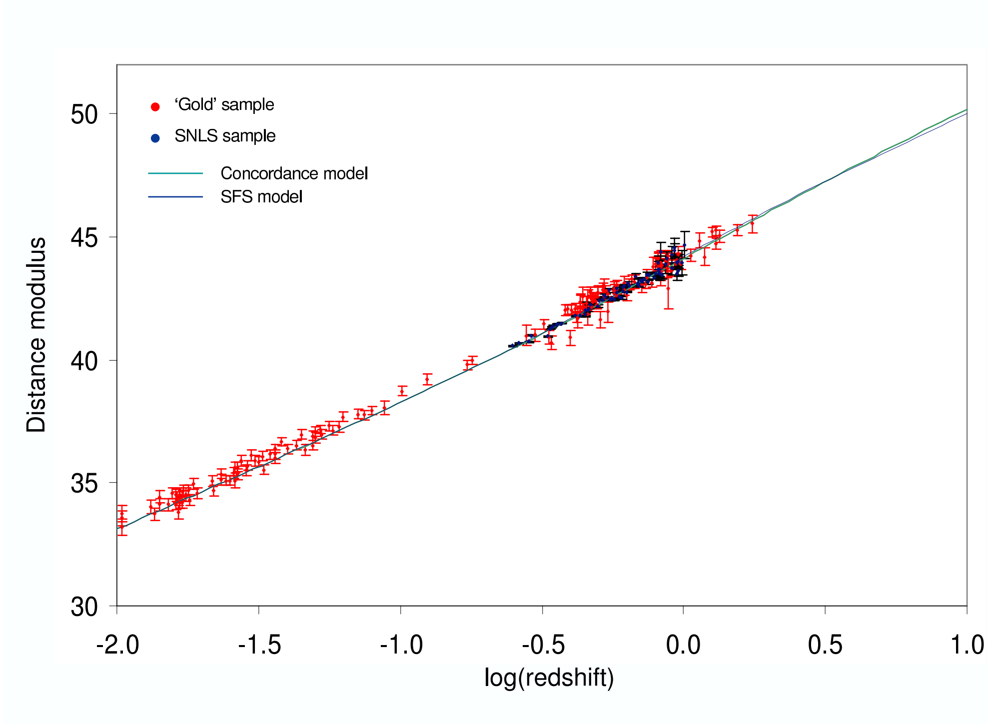
where r_1 comes from null geodesic equation

$$\int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}} = \int_{t_1}^{t_0} \frac{cdt}{a(t)} = ct_s \int_{y_0}^{y_1} \frac{dy}{a(y)} = \frac{c}{H_0 a_0} \int_0^z \frac{dz}{E(z)} , \tag{27}$$

and E(z) cannot be given explicitly here as in standard cosmology, and must be calculated numerically. The redshift is

$$1 + z = \frac{a(t_0)}{a(t_1)} = \frac{\delta + (1-\delta)\, y_0^m - \delta\,(1-y_0)^n}{\delta + (1-\delta)\, y_1^m - \delta\,(1-y_1)^n} , \tag{28}$$

SFS dark energy versus Λ-term dark energy (concordance cosmology - CC)



Distance modulus $\mu_L = m - M$ for the CC model ($H_0 = 72 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$, $\Omega_{m0} = 0.26$, $\Omega_{\Lambda 0} = 0.74$) (dashed curve) and SFS model ($m = 2/3 = 0.6666, n = 1.9999, \delta = -0.471, y_0 = 0.99936$) (solid curve). Open circles are for the 'Gold' data and filled circles are for SNLS data.

Exotic-singularity-driven dark energy surprise.

Surprising remark:
If the age of the SFS model is equal to the age of the CC model, i.e. $t_0 = 13.6$ Gyr, one finds that **an SFS is possible in only 8.7 million years!!!**.

- In this context it is no wonder that the singularities were termed “sudden”.
- It was checked that GSFS (generalized SFS - no energy conditions violation) are always more distant in future. That means the strongest of SFS type singularities is more likely to become reality.
- A practical tool to recognize them well in advance is to measure possible large values of statefinders (deceleration parameter, jerk, snap etc.)!

Interesting point: SFS and other exotic singularities plague loop quantum cosmology! - see Wands et al. PRL ’08 (arXiv: 0808.0190); Singh and Vidotto 1012.1307).

Big-Brake-exotic-singularity-driven dark energy.

SFS with $a = a_b$ = const., $\dot{a} = 0$ ($\varrho \to 0$), and $\ddot{a} \to -\infty$ ($p \to \infty$) were also termed Big-Brake (Gorini, Kamenschchik et al. PRD 69 (2004), 123512). They fulfill an anti-Chaplygin gas equation of state of the form

$$p = \frac{A}{\varrho} \qquad \mathrm{A} = \mathrm{const.} \quad . \tag{29}$$

They were studied in the context of the tachyon cosmology by Keresztes Gergely et al. PRD 79, 083504 (2009), Gergely, Keresztes, Gorini, Kamenschchik, Polarski 1009.0776.

However, due to the imposition of different values of parameters which are given by tachyon constraints (plus anti-Chaplygin gas constraints) the closest singularity in their model appears

– 1 Gyr in future

– and the furthest even 44 Gyr in future.

Despite, they of course can serve as a source of dark energy.

Big-Brake is just an SFS.

Big-Brake, is achieved for $\varrho \to 0$ and $p \to \infty$ in the anti-Chaplygin gas model

$$p(t) = \frac{A}{\varrho(t)} \qquad (A \geq 0) \ . \tag{30}$$

Consider the first time derivative of the SFS scale factor:

$$\dot{a}(t) = a_s \left[\frac{m(1-\delta)}{t_s} y^{m-1} + \delta \frac{n}{t_s} \left(1-y\right)^{n-1} \right] \ . \tag{31}$$

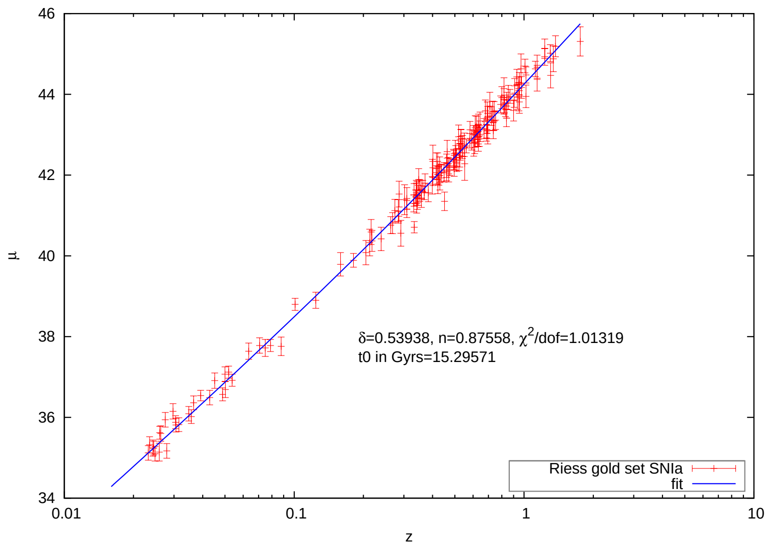
Requiring that $\dot{a} \to 0$, which corresponds to $\varrho \to 0$ at $y = 1$ we have a condition that either $m = 0$ or $\delta \to 1$. In fact, these conditions are almost equivalent since

$$\lim_{m \to 0} a(y) = a_s[1 - \delta(1-y)^n] \ , \tag{32}$$

$$\lim_{\delta \to 1} a(y) = a_s[1 - (1-y)^n] \ , \tag{33}$$

though the first one does not restrict δ (and also it has a standard Friedmann limit $\delta \to 0$ - a static one).

FSF (type III) v. supernovae



We have preliminary found that even the type III (Finite Scale Factor) singularity can be closer than this, i.e.

$t_s - t_0 \approx 0.3 Gyrs$ (about 30 times larger than the time to an SFS)

with the choice of parameters to be:

$\ddot{a} > 0$ for $\delta > 0$:

$n = 0.87558$; δ = 0.53938; t_0 = 15.29571 Gyrs

CMB shift parameter.

It is possible to fit other tests but at the expense of relaxing the range of the parameter m which refers to Big-Bang limit ($m = 2/3$ is dust).
Shift parameter is:

$$\mathcal{R} = \frac{l_1'^{TT}}{l_1^{TT}} \tag{34}$$

where
l_1^{TT} – the temperature perturbation CMB spectrum multipole of the first acoustic peak in SFS model
$l_1'^{TT}$ – the multipole of a reference flat standard Cold Dark Matter model.
One usually uses a rescaled shift parameter:

$$\mathcal{R} = \frac{H_0 a_0}{c}\sqrt{\Omega_{m0}} r_{dec} = \sqrt{\Omega_{m0}} a'(y) \int_{y_{dec}}^{y_0} \frac{dy}{a(y)} = \sqrt{\Omega_{m0}} \int_0^{z_{dec}} \frac{dz}{E(z)}, \tag{35}$$

and WMAP data gives $\mathcal{R} = 1.70 \pm 0.03$ (Wang et al. 2006).

Baryon acoustic oscillations.

The Alcock-Paczyński effect says that one is able to calculate the distortion of a spherical object in the sky without knowing its true size.
This can be done by measuring its transverse extend (using the angular diameter distance $d_A = l/\Delta\theta$, where l is the linear size of an object) and line-of-sight extend (using the redshift distance $\Delta x = c\Delta t/a(t) = ct_s \Delta y/a(y)$) (see e.g. Nesseris et al. 2006). As a result one defines the volume distance as

$$D_V^3 = d_A^2 \Delta x \quad , \tag{36}$$

so that one has

$$D_V = \left[\left(\int_{y_1}^{y_0} \frac{ct_s dy}{a(y)} \right)^2 \left(\frac{ct_s \Delta y}{a(y)} \right) \right]^{\frac{1}{3}} = \left[\left(\frac{c}{a_0 H_0} \int_0^z \frac{dz}{E(z)} \right)^2 \left(\frac{c}{a_0 H_0} \frac{\Delta z}{E(z)} \right) \right]^{\frac{1}{3}}$$

Eisenstein et al. (2005) gave $D_V(\Delta z = z_{BAO} = 0.35) = 1370 \pm 64$ Mpc (an acoustic peak for 46748 luminous red galaxies (LRG) selected from the SDSS (Sloan Digital Sky Survey).

Baryon acoustic oscillations - dimensionless parameter A.

For our sudden future singularity model (14) it is more convenient to use a dimensionless quantity $\mathcal{A}$ which is obtained multiplying D_V by $\sqrt{\Omega_{m0}}/(ct_s z_{BAO})$ or by $\sqrt{\Omega_{m0}}(a_0 H_0)/(cz_{BAO})$ to get

$$\mathcal{A} = \sqrt{\Omega_{m0}} a'(y_0) \left[\frac{a(y_{BAO})}{a'(y_{BAO}) a(y_0)} \right]^{\frac{1}{3}} \left[\frac{1}{z_{BAO}} \int_{y_{BAO}}^{y_0} \frac{dy}{a(y)} \right]^{\frac{2}{3}} \tag{38}$$

or

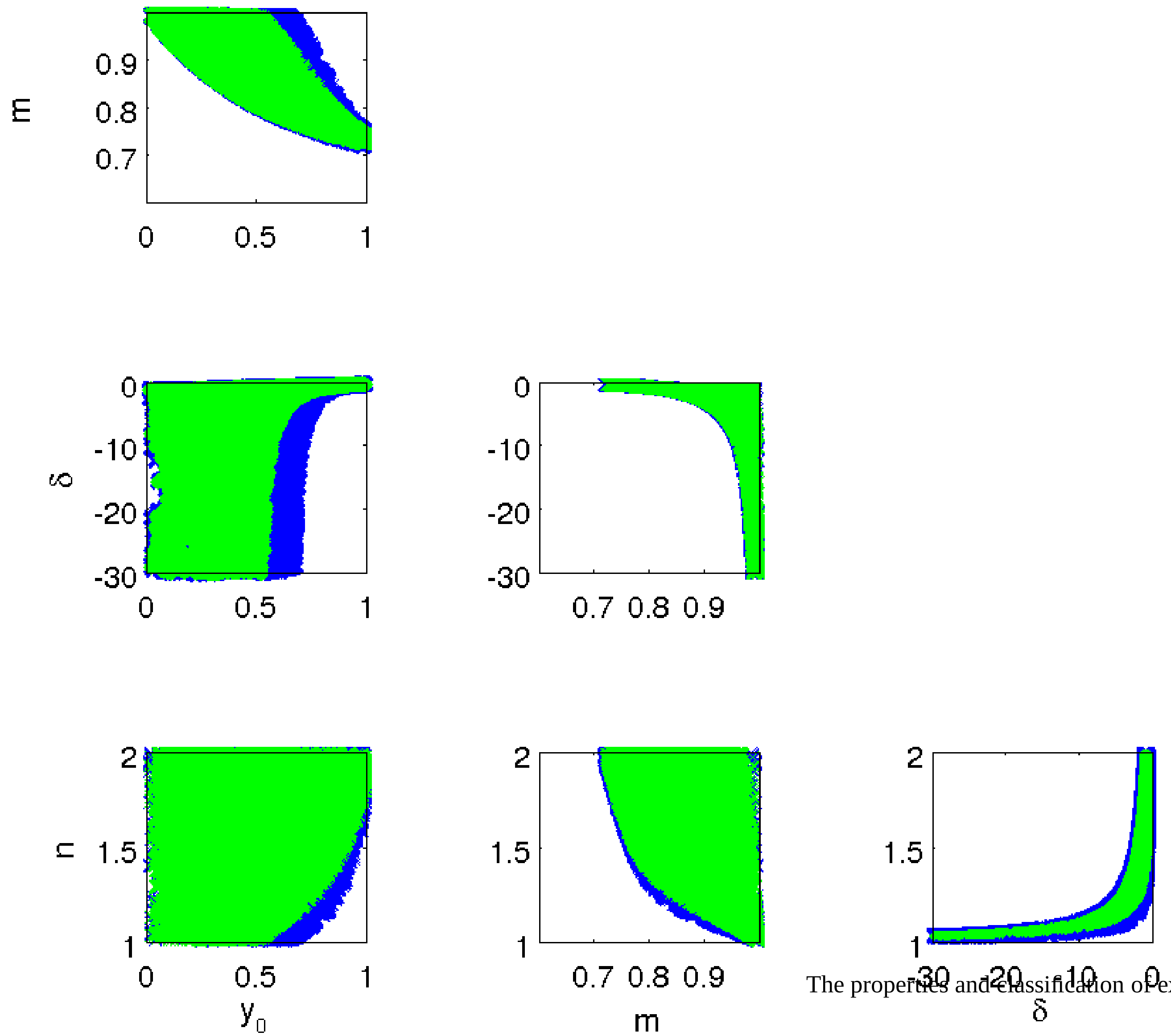
$$\mathcal{A} = \sqrt{\Omega_{m0}} E(z_{BAO})^{-1/3} \left[\frac{1}{z_{BAO}} \int_0^{z_1} \frac{dz}{E(z)} \right]^{2/3} \tag{39}$$

It should have the value (Eisenstein et al. 2005)

$$\mathcal{A} = 0.469 \left(\frac{n}{0.98} \right)^{-0.35} \pm 0.017 \ , \tag{40}$$

where n is the spectral index (now taken about ~ 0.96).

Combined bound: supernovae, CMB shift parameter and BAO.



Combined bound: conclusions.

There is a region for the 3 tests to overlap but it requires that in the near-to-Big-Bang phase the dominating fluid has slightly negative pressure

$$m \approx 0.72 \qquad \rightarrow \qquad w \approx -0.083 \tag{41}$$

6. The universe through an exotic singularity - averaging.

One is able to construct a hybrid model which allows Big-Bang, Sudden Future Singularity and finally Big-Crunch given by:

$$a_L(t) = a_s \left[\delta + \left(1 + \frac{t}{t_B}\right)^m (1-\delta) - \delta \left(-\frac{t}{t_B}\right)^n\right] \tag{42}$$

with $t_B < 0$ - the Big-Bang time, and $t = 0$ and SFS time;

$$a_R(t) = a_s \left[\delta + \left(1 - \frac{t}{t_C}\right)^m (1-\delta) - \delta \left(\frac{t}{t_C}\right)^n\right] \tag{43}$$

with $t_C > 0$ - the Big-Crunch time. In the high pressure regime $t \to 0$ these are approximated by

$$a_L \approx a_s \left[1 + \frac{m}{t_B}(1-\delta)\, t\right], \tag{44}$$

$$a_R \approx a_s \left[1 - \frac{m}{t_C}(1-\delta)\, t\right]. \tag{45}$$

Spacetime averaging of the open universes

A.K. Raychaudhuri (PRL 80, 654 (1998) proposed that one may average physical and kinematical scalars over the whole open spacetime provided they vanish rapidly at spatial and temporal infinity as follows

$$< \chi >= \lim_{x^a \to \infty} \frac{\int \int \int \int_{-x^a}^{x^a} \chi \sqrt{-g} d^4x}{\int \int \int \int_{-x^a}^{x^a} \sqrt{-g} d^4x} \tag{46}$$

By an open model it is meant that the ratio of the 3-volume hypersurfaces to a 4-volume of spacetime vanishes, i.e.,

$$\frac{\int \int \int \sqrt{|^3 g|} d^3x}{\int \int \int \int \sqrt{-g} d^4x} = 0. \tag{47}$$

His idea was to tight the vanishing of the average $< \chi >$ with the singularity avoidance in cosmology.

Spacetime averaging - density and pressure.

For the pressure, the energy density, and the average acceleration we have

$$< p >= - \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} \frac{\int_{t_0}^{t_1} a^3 \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt} \tag{48}$$

and

$$< \varrho >= \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} \frac{3 \int_{t_0}^{t_1} a^3 \left(\frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt} . \tag{49}$$

$$< \dot{\theta} >= \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} \frac{3 \int_{t_0}^{t_1} a^3 \left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt} . \tag{50}$$

Spacetime averaging - standard and phantom models

$$< p >_{stand} = \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} -\frac{4}{\gamma}\left(\frac{1}{\gamma} - 1\right) \frac{\int_{t_0}^{t_1} t^{2\left(\frac{1}{\gamma}-1\right)} dt}{\int_{t_0}^{t_1} t^{\frac{2}{\gamma}} dt} \longrightarrow 0,$$

$$< \varrho >_{stand} = \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} -\frac{4}{3\gamma^2} \frac{\int_{t_0}^{t_1} t^{2\left(\frac{1}{\gamma}-1\right)} dt}{\int_{t_0}^{t_1} t^{\frac{2}{\gamma}} dt} \longrightarrow 0$$

$$< p >_{ph} = \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} -\frac{4}{\mid \gamma \mid}\left(\frac{1}{\mid \gamma \mid} + 1\right) \frac{\int_{t_0}^{t_1} t^{-2\left(\frac{1}{|\gamma|}+1\right)} dt}{\int_{t_0}^{t_1} t^{-\frac{2}{|\gamma|}} dt} \longrightarrow \infty,$$

$$< \varrho >_{ph} = \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} -\frac{4}{3\gamma^2} \frac{\int_{t_0}^{t_1} t^{-2\left(\frac{1}{|\gamma|}+1\right)} dt}{\int_{t_0}^{t_1} t^{-\frac{2}{|\gamma|}} dt} \longrightarrow \infty$$

Spacetime averaging - SFS and FSF models

$$\dot{a}_L(t) = a_s \left[\frac{m}{t_B} \left(1 + \frac{t}{t_B} \right)^{m-1} (1-\delta) + \delta \frac{n}{t_B^n} (-t)^{n-1} \right] \tag{51}$$

$$\dot{a}_R(t) = a_s \left[-\frac{m}{t_C} \left(1 - \frac{t}{t_C} \right)^{m-1} (1-\delta) + \delta \frac{n}{t_C^n} (t)^{n-1} \right] \tag{52}$$

$$\frac{\ddot{a}_L}{a_s} = \frac{m(m-1)(1-\delta)}{t_B^2} \left(1 + \frac{t}{t_B} \right)^{m-2} - \frac{\delta n(n-1)}{t_B^n} (-t)^{n-2} \tag{53}$$

$$\frac{\ddot{a}_R}{a_s} = \frac{m(1-m)(1-\delta)}{t_C^2} \left(1 - \frac{t}{t_C} \right)^{m-2} + \frac{\delta n(n-1)}{t_C^n} t^{n-2} \tag{54}$$

Only the last terms blow up to give infinite pressure for $1 < n < 2$ at $t = 0$ so that we neglect other terms in a, $\dot{a}$ and $\ddot{a}$.

Spacetime averaging - SFS and FSF models

$$< \dot{\theta} >_{SFS,L} = \lim_{\substack{t_0 \to -t_B \\ t_1 \to 0}} -3n \frac{\int_{t_0}^{t_1} (-t)^{3n-2} dt}{\int_{t_0}^{t_1} (-t)^{3n} dt} \tag{55}$$

$$= \lim_{\substack{t_0 \to -t_B \\ t_1 \to 0}} -3n \frac{3n+1}{3n-1} \frac{(-t_1)^{3n-1} - (-t_0)^{3n-1}}{(-t_1)^{3n+1} - (-t_0)^{3n+1}} \to \frac{1}{t_B^2}$$

$$< \dot{\theta} >_{SFS,R} = \lim_{\substack{t_0 \to 0 \\ t_1 \to t_C}} 3n \frac{\int_{t_0}^{t_1} t^{3n-2} dt}{\int_{t_0}^{t_1} t^{3n} dt} \tag{56}$$

$$= \lim_{\substack{t_0 \to 0 \\ t_1 \to t_C}} 3n \frac{3n+1}{3n-1} \frac{t_1^{3n-1} - t_0^{3n-1}}{t_1^{3n+1} - t_0^{3n+1}} \to \frac{1}{t_C^2}$$

These averages are finite for SFS, but they may blow up for FSF if $0 < n < 1/3$!

Subtleties

- BB, BC singularities - all the energy conditions fulfilled, averages vanish
- BR singularity - no EC fulfilled, averages blow up
- SFS - only dominant energy violated, averages finite
- It seems that BR is stronger singularity that BB, BC on the ground of averaging.
- SFS is weaker, but FSF does not seem so.

7. Summary

- Exotic singularities can be related to new physical sources of gravity serving as dark energy.
- First example source - phantom - produces an exotic singularity – a Big-Rip in which ($a \to \infty$ and $\varrho \to \infty$) which is different from a Big-Bang/Crunch.
- Investigations of phantom inspired other searches for non-standard singularities (sudden future, generalized sudden future (=Big-Brake), type III (Finite Scale Factor), type IV (Big-Separation), $w-$singularities etc.) which, in fact, are not necessarily the "true" singularities (according to Hawking and Penrose definition), as sources of dark energy.
- Exotic singularities are, in fact, motivated by fundamental theories of particle physics (scalar-tensor, superstring, brane, loop quantum cosmology etc.).

summary contd.

- Big-Rip which serves as dark energy despite it may happen in 20 Gyr, while weak singularities (of tidal forces and their derivatives) may serve as dark energy if they are quite close in the near future. For example an SFS may even appear in 8.7 Myr with no contradiction with data. A GSFS always appears later. Type III (FSF) is possible in about 0.3 Gyr. Finally, a Big-Brake (which is also an SFS) in tachyon cosmology context is at least 1 Gyr away from now.
- An SFS universes can be fitted to SnIa, CMB and BAO data but at the expense of admitting an approach to a Big-Bang by a fluid which is not exactly dust (m=0.66) but has a slightly negative pressure (m = 0.73 and so w = -0.09).
- Weak exotic singularities (e.g. SFS) allow extended objects to go through them - this allows construction of a hybrid models of the universe in which weak singularities are only an episode between the strong singularities such as Big-Bang or Big-Rip.