Are the singularities limits of cosmology?

Mariusz P. Dąbrowski

Institute of Physics, University of Szczecin, Poland

and

Copernicus Center for Interdisciplinary Studies, Kraków, Poland

"The limits of physics and cosmology", Kraków - 9 May 2014

Plan:

- 1. Singularities.
- 2. Big-Bang and non-Big-Bang singularities in Friedmann cosmology.
- 3. Classification and observational verification.
- 4. The evolution through a singularity - averaging approach.
- 5. Varying constants removing or changing singularities.
- 6. Conclusions.

1. Singularities

- Singularities $\approx$ infinities of physical/mathematical quantities.
- Experimental/observational problem - one is not able to measure an infinite quantity.
- This is why we say that our (whatever) the theory fails.
- Appear in physics and we have to deal with them.
- They appear in Newton's theory (e.g. when spherical shell collapses to a point at $r = 0$).
- We experience them also in general relativity (best-known examples: Big-Bang $t = 0$, black hole $r = 0$ singularity - collapsing as a result of gravitational attraction).

Definition(s) - what is a singularity?

- Intuitively: “place” in which some “pathology” (infinity, blow-up, etc.) is observed.
- Physical fields (like the electric field) can be singular but in general relativity we deal with spacetime itself.
- Physical field singularities and spacetime singularities are independent (e.g. string theory).
- Various definitions have been proposed based on the blow-up of the curvature tensor and curvature invariants, differentiating them as “boundaries” of spacetimes (there are some “pathological” examples: gravitational waves, conical singularity, etc.).
- The matter is very subtle - the best definition is considered to be geodesic incompletness (e.g. Wald 1983) which allows to practically detect them without “adopting” them to the theory.

Definition(s) - what is a singularity?

- Spacetime is singular if there exists **at least one geodesic which is incomplete** i.e. which cannot be extended in at least one direction and has only a finite range of affine parameter (proper time or length for non-null geodesics).
- This definition seems to give the limits of general relativity and especially cosmology. “Points”, “regions”, “holes” which are not reachable by our physical theory.
- This is a kind of “minimalistic” approach which does not tell us the nature of these singularities: e.g. how they influence the physical and geometrical quantities - I will try to get into more subtleties of the matter applying some other definitions to differentiate them.

The strength of singularities.

- Tipler's (Phys. Lett. A64, 8 (1977)) definition (of a strong singularity): $I^i_j(\tau) = \int_0^\tau d\tau' \int_0^{\tau'} d\tau'' |R^i_{ajb} u^a u^b|$ diverges on the approach to a singularity at $\tau = \tau_s$
- i.e. an extended object is crushed to zero volume (represented by three linearly independent, vorticity-free geodesic deviation vectors at p parallely transported along causal geodesic l) at the singularity by infinite tidal forces
- Królak's (CQG 3, 267 (1988)) definition (of a strong singularity): $I^i_j(\tau) = \int_0^\tau d\tau' |R^i_{ajb} u^a u^b|$ diverges on the approach to a singularity at $\tau = \tau_s$
- i.e. the expansion of every future-directed congruence of null (timelike) geodesics emanating from point p and containing l becomes negative somewhere on l
- For null geodesics one replaces Riemann by the Ricci tensor components.

Geodesics and geodesic deviation.

- To comply with geodesic incompleteness definition we will use geodesic equation

$$\frac{d^2x^\alpha}{d\lambda^2} + \Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{d\lambda}\frac{dx^\gamma}{d\lambda} \tag{1}$$

  where λ is an affine parameter.

- Also use geodesic deviation equation

$$\frac{D^2n^\alpha}{d\lambda^2} + R^\alpha{}_{\beta\gamma\delta}u^\beta n^\gamma u^\delta = 0\ , \tag{2}$$

  with n^α being the deviation vector separating neighboring geodesics.

- We also use the so-called Raychaudhuri spacetime averaging.

Energy conditions.

- S(trong) E(nergy) C(ondition) (attractivity of gravity)

$$\varrho c^2 + 3p \geq 0, \qquad \varrho c^2 + p \geq 0\ . \tag{3}$$

- N(ull) E(nergy) C(ondition) (non-spacelike flux)

$$\varrho c^2 + p \geq 0 \tag{4}$$

- W(eak) E(nergy) C(ondition) (positivity of energy)

$$\varrho c^2 + p \geq 0, \rho c^2 \geq 0 \tag{5}$$

- D(ominant) E(nergy) C(ondition) (pressure smaller than energy density)

$$\mid p \mid \leq \varrho c^2, \varrho c^2 \geq 0 \tag{6}$$

2. Big-Bang and non-Big-Bang singularities in Friedmann cosmology.

We will restrict ourselves to the simple framework of Einstein-Friedmann cosmology based on the **two equations for three unknown functions** of time $a(t), p(t), \varrho(t)$

$$\varrho = \frac{3}{8\pi G}\left(\frac{\dot{a}^2}{a^2} + \frac{Kc^2}{a^2}\right) , \tag{7}$$

$$p = -\frac{c^2}{8\pi G}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{Kc^2}{a^2}\right) . \tag{8}$$

They give standard Big-Bang/Crunch singularities once an equation of state, e.g., of a barotropic type ($w =$ const. ≥ -1) is assumed:

$$p(t) = w\varrho(t) \qquad \rightarrow a(t) \propto t^{\frac{2}{3(w+1)}} . \tag{9}$$

Big-Bang/Big-Crunch - BB, BC

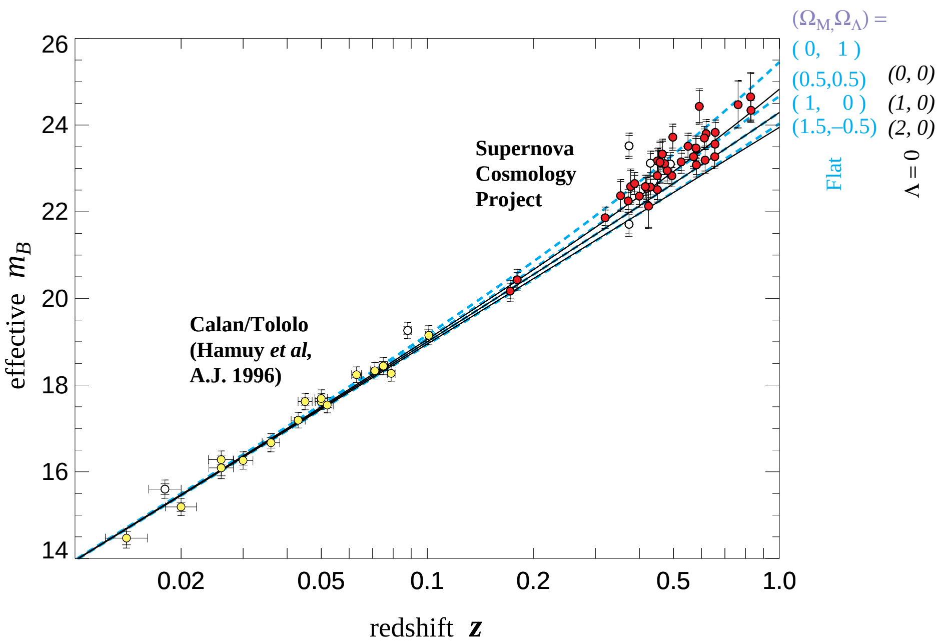
Comply with the definition of being geodesically incomplete.

Big-Bang singularity has: $a \to 0$, $\varrho, p \to \infty$ (curvature "blow-up").

(Big-Crunch) ($K = +1$ only) has: $a \to 0$, $\varrho, p \to \infty$

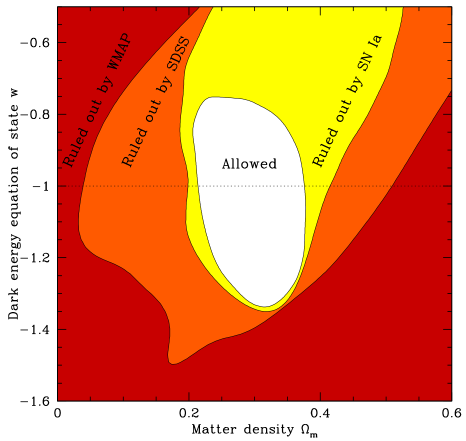
Asymptotic emptiness ($K = 0, -1$) continue to: $\varrho, p \to 0$ for $a \to \infty$.

First supernovae observations (Perlmutter et al. 1999), despite SEC violation due to Λ-term, did not influence their idea:



Non-Big-Bang singularities in cosmology.

However, WMAP + SDSS + Supernovae **combined bound on the dark energy barotropic index** w (e.g. Tegmark et al. (2004); Shafer & Huterer 1312.1668)



- **allow that the dark energy with $p < -\varrho$ (phantom) can be admitted!** which leads to
- **cosmic "no-hair" theorem violation - even a small fraction of phantom dark energy may dominate the evolution $\rightarrow$ Big-Rip singularity**
- NEC, WEC, DEC violated!

Big-Rip (BR - type I) as non-Big-Bang singularity.

Since for phantom $w < -1$, then for convenience we may take

$$| w + 1 | = -(w + 1) > 0 \ , \tag{10}$$

so $a(t) = t^{-2/3|w+1|}$ and the conservation law for phantom gives

$$\varrho \propto a^{3|w+1|} \ . \tag{11}$$

- Conclusion: the bigger the universe grows, the denser it is, and **it becomes dominated by phantom (which overcomes Λ-term)** – an exotic future singularity appears – Big-Rip $\varrho, p \to \infty$ for $a \to \infty$
- Only for $-5/3 < w < -1$ the null geodesics are geodesically complete; for other values of w, including all timelike geodesics, there is a geodesic incompleteness (Fernandez-Jambrina and Lazkoz, gr-qc/0607073).
- Curvature invariants R^2, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ diverge.

Big-Rip approach:

In a Big-Rip scenario everything is pulled apart on the approach to a Big-Rip in a reverse order (Caldwell et al. PRL '03).
Specifically, for $w = -3/2$ Big-Rip will happen in 20 Gyr from now and the scenario will be as follows:

- in 1 Gyr before BR - clusters are erased
- in 60 Myr before BR - Milky Way is destroyed
- 3 months before BR - Solar System becomes unbound
- 30 min before BR - Earth explodes
- 10^{-19} s before BR - atoms are dissociated
- nuclei etc. .....

Sudden Future Singularity (SFS - type II) as non-BB singularity.

Surprise of a Big-Rip gave a push to studies some other types of singularities as possible sources of dark energy

Barrow (2004) (based on earlier study of Barrow, Galloway, Tipler 1986) dropped an assumption about the imposition of the equation of state (3)

$$p \neq p(\varrho), \tag{12}$$

but assumed an analytic form of the scale factor instead:

$$a(t) = a_s \left[\delta + (1-\delta)\, y^m - \delta\, (1-y)^n\right] , \qquad y \equiv \frac{t}{t_s} \tag{13}$$

where $a_s \equiv a(t_s) = \text{const.}$ and $\delta, m, n = \text{const.}$

$$\dot{a} = a_s \left[\frac{m}{t_s}(1-\delta)\, y^{m-1} + \delta \frac{n}{t_s}(1-y)^{n-1}\right] , \tag{14}$$

$$\ddot{a} = \frac{a_s}{t_s^2}\left[m\,(m-1)\,(1-\delta)\, y^{m-2} - \delta n\,(n-1)\,(1-y)^{n-2}\right] . \tag{15}$$

Sudden Future Singularity ...

Provided

$$1 < n < 2, \tag{16}$$

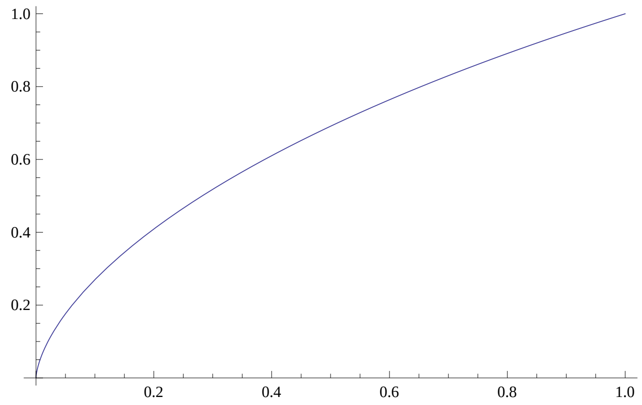
one gets a Big-Bang at $t = 0$ as well as a new type of singularity at $t = t_s$ - a Sudden Future Singularity (SFS) (or type II - Nojiri, Odintsov, Tsujikawa 2005) which:

– manifests as a singularity of pressure p (or $\ddot{a}$) only

– leads to the **dominant energy condition violation only.** In fact we have:

$$a = \text{const.}, \qquad \dot{a} = \text{const.} \qquad \varrho = \text{const.}$$
$$\ddot{a} \to -\infty \qquad p \to \infty \qquad \text{for} \qquad t \to t_s \tag{17}$$

Sudden Future Singularities ...



Interesting observations:

Schwarzschild horizon at $r = r_g$ - metric singular, curvature invariants regular,

Sudden Future Singularity at $t = t_s$ - metric regular, curvature invariants diverge.

commonly appear in Loop Quantum Cosmology (LQC) (Cailletau et al. PRL 101, 251302 (2008)) on the contrary to big-bang and big-rip which are avoided in LQC (e.g. Sami et al. gr-qc/0605113).
This is due to quadratic term in the Friedmann equation (e.g. Bojowald PRL '02, gr-qc/0601085) which simulates negative brane tension (extra timelike dimensions - e.g. Shtanov and Sahni PLB, 557 (2003), 1):

$$H^2 = \frac{1}{3m_{pl}^2}\left(\rho - \frac{\rho^2}{\rho_c}\right) - \frac{k}{a^2} \ , \tag{18}$$

where the critical density is

$$\rho_c \equiv \frac{\sqrt{3}}{16\pi\gamma^3 G^2 \hbar^2} \ , \tag{19}$$

and γ is the Barbero-Immirzi parameter ($\gamma \approx 0.2375$, Meissner gr-qc/0407052).

Generalized Sudden Future singularities (type IIg).

Sudden Future Singularities may be generalized to GSFS if we take a general scale factor time derivative of an order r:

$$\begin{aligned} a^{(r)} &= a_s \left[\frac{m(m-1)...(m-r+1)}{t_s^r} \left(1-\delta\right) y^{m-r} \right. \\ &+ \left. (-1)^{r-1} \delta \frac{n(n-1)...(n-r+1)}{t_s^r} \left(1-y\right)^{n-r} \right] , \end{aligned} \tag{20}$$

and choose (Barrow 2004, Lake 2004) $r-1 < n < r$. Then for any integer r we have a singularity in the scale factor derivative $a^{(r)}$, and consequently in the appropriate pressure derivative $p^{(r-2)}$.

None of the energy conditions (EC) is violated for $r \geq 3$!!!

Finite Scale Factor (FSF - type III).

The new exotic singularities were found as Type III singularities which we will call Finite Scale Factor - FSF singularities are characterized by the following conditions (Nojiri, Odintsov, Tsujikawa 2005):

$$a = a_s = \text{const.},\ \varrho, \dot{a}_s \rightarrow \infty,\ |p|, \ddot{a}_s \rightarrow \infty$$

The simplest way to get them is to apply the scale factor as given previously for SFS, i.e.,

$$a(t) = a_s \left[\delta + (1-\delta)\, y^m - \delta\, (1-y)^n\right] , \qquad y \equiv \frac{t}{t_s} \tag{21}$$

where $a_s \equiv a(t_s) =$ const. and $\delta, A, m, n =$ const., but with the range of parameter n changed from $1 < n < 2$ onto

$$0 < n < 1$$

Big Separation - BS (type IV)

Type IV singularity is when (Nojiri, Odintsov, Tsujikawa 2005):

$$a = a_s = \text{const.}, \varrho \to 0, p \to 0, \dot{p}, \dddot{a}, \ddot{H} \to \infty \text{ etc.}$$

and so it is similar to Generalized Sudden Future singularity with only one exception: it also gives the divergence of the barotropic index in the barotropic equation of state

$$p(t) = w(t)\varrho(t)$$

i.e.,

$$w(t) \to \infty$$

Barotropic index $w-$singularity (Type V)

Assuming the following type of scale factor (MPD, Denkiewicz 2009):

$$
\begin{aligned}
a(t) &= \frac{a_s}{1-\frac{3\gamma}{2}\left(\frac{n-1}{n-\frac{2}{3\gamma}}\right)^{n-1}} \\
&+ \frac{1-\frac{2}{3\gamma}}{n-\frac{2}{3\gamma}} \frac{n a_s}{1-\frac{2}{3\gamma}\left(\frac{n-\frac{2}{3\gamma}}{n-1}\right)^{n-1}} \left(\frac{t}{t_s}\right)^{\frac{2}{3\gamma}} \\
&+ \frac{a_s}{\frac{3\gamma}{2}\left(\frac{n-1}{n-\frac{2}{3\gamma}}\right)^{n-1}-1} \left(1-\frac{1-\frac{2}{3\gamma}}{n-\frac{2}{3\gamma}}\frac{t}{t_s}\right)^{n}, \qquad (22)
\end{aligned}
$$

with the admissible values of the parameters: $\gamma = w + 1 > 0$ and $n \neq 1$.

$w-$singularity

one gets a blow-up of the effective barotropic index, i.e.,

$$w(t_s) = \frac{1}{3}\left[2q(t_s) - 1\right] \to \infty \; , \tag{23}$$

accompanied by

$$p(t_s) \to 0; \quad \varrho(t_s) \to 0 \; . \tag{24}$$

There is a kind of duality between the Big-Bang and the w-singularity in the form

$$p_{BB} \leftrightarrow \frac{1}{p_w}, \quad \varrho_{BB} \leftrightarrow \frac{1}{\varrho_w}, \quad w_{BB} \leftrightarrow \frac{1}{w_w} \; . \tag{25}$$

Physically real: w-sing. appear in $f(R)$ gravity (Starobinsky ’80), scalar field gravity (Setare, Saridakis ’09), and brane gravity (Sahni, Shtanov ’05)

3. Classification and observational verification.

- Type 0 - Big-Bang (Big-Crunch) $a \to 0$, $p \to \infty$, $\varrho \to \infty$
- Type I - Big-Rip $a(t_s) \to \infty$ ($t_s < \infty$), $p \to \infty$, $\varrho \to \infty$ (Caldwell 2002)
- Type II - Sudden Future (includes Big Boost and Big-Brake) $a(t_s) =$ const., $\varrho =$ const., $p \to \infty$ (Barrow 2004)
- Type IIg - Generalized Sudden Future $a(t_s)$= const., $\varrho =$ const., $p =$const., $\dddot{a} \to \infty$ etc., $w < \infty$ (Barrow 2004)
- Type III - Finite Scale Factor (also Big-Freeze) $a(t_s)$ = const., $\varrho \to \infty$, $p \to \infty$ (NOT 2005, Denkiewicz 2012)
- Type IV - Big Separation: $a(t_s)$= const., $p = \varrho = 0$, $w \to \infty$, $\dddot{a} \to \infty$ etc. (NOT 2005) (and generalizations $p = \varrho =$const. Yurov 2010)
- Type V - w-singularity $a(t_s)$= const., $p = \varrho = 0$, $w \to \infty$ (MPD, Denkiewicz 2009) (and generalizations $p =$const. Yurov 2010)
- More subtleties: Little-Rip $a(t_s) \to \infty$, $\varrho(t_s) \to \infty$ ($t_s \to \infty$) and Pseudo-Rip $\varrho(t_s) < \infty$ ($t_s \to \infty$) (Frampton et al. 2011, 2012)

Non-BB singularities and geodesic incompleteness - no limit of cosmology.

The application of geodesic equation shows that some non-BB singularities can be continued through since geodesics do not feel them at all - they are not singular there since $a_s = a(t_s) =$ const. (Fernandez-Jambrina, Lazkoz PRD 74, 064030 (2006)) - **no geodesic incompleteness**

$$\left(\frac{dt}{d\tau}\right)^2 = A + \frac{P^2 + KL^2}{a^2(t)} , \quad (26)$$

$$\frac{dr}{d\tau} = \frac{P_1 cos\phi + P_2 \sin\phi}{a^2(t)}\sqrt{1 - Kr^2} , \quad (27)$$

$$\frac{d\phi}{d\tau} = \frac{L}{a^2(t)r^2} . \quad (28)$$

However, geodesic deviation equation (i.e. a bunch of geodesics)

$$\frac{D^2 n^\alpha}{d\lambda^2} + R^\alpha_{\ \beta\gamma\delta} u^\beta n^\gamma u^\delta = 0 , \quad (29)$$

do feel SFS since at $t = t_s$ we have the Riemann tensor $R^\alpha_{\ \beta\gamma\delta} \to \infty$.

“Go through” singularities.

Specifically, for SFS we have

$$R^{\alpha}_{\ 0\beta 0} = -\frac{\ddot{a}}{a}\delta^{\alpha}_{\beta} \tag{30}$$

which for integral curves of $u = \partial/\partial t$ (geodesics with an affine parameter t) gives

$$\dot{u}^{\alpha} = -R^{\alpha}_{\ 0\beta 0}n^{\beta} \propto \ddot{a} \tag{31}$$

which diverges to $-\infty$ at $t = t_s$.
Physically it means that the tidal forces manifest as the (infinite) force which reverses (stops) the increase of separation of geodesics but the geodesics themselves can evolve further - the universe can continue its evolution through a singularity.
It is like a turning point for a harmonic oscillator.

Extended objects through non-BB singularities.

This manifests nicely if one considers an extended objects falling into such a non-BB singularity such as a classical string (Polyakov type) given by the action (Balcerzak and MPD 2006)

$$S = -\frac{T}{2}\int d\tau d\sigma \eta^{ab} g_{\mu\nu}\partial_a X^{\mu}\partial_b X^{\nu} \tag{32}$$

The invariant string size $S(\tau) = 2\pi a(\eta(\tau))R(\tau)$ (circular ansatz with radius R) shows that they are: infinitely stretched $S \to \infty$ at Big-Rip (string is destroyed) while for SFS the scale factor is finite at η-time at SFS so that the invariant string size is also finite. The same is true for type III, IV and GSFS.

This means that strings **not destroyed** at such singularities.

Strong or weak - classification of non-BB singularities.

Fernandez-Jambrina (PRD 82, 124004 (2010)) used Puiseux series expansion

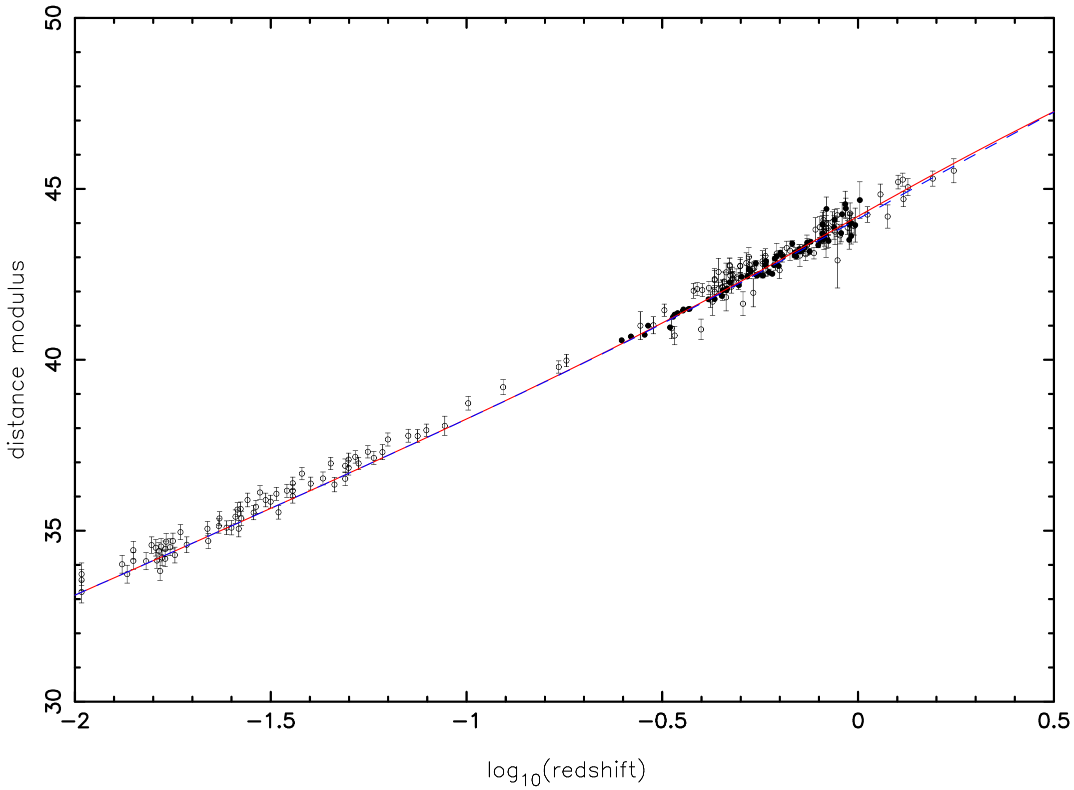
$$a(t) = c_0 + (t_s - t)^{\eta_0} + c_1 (t_s - t)^{\eta_1} + c_2 (t_s - t)^{\eta_2} + \ldots \qquad \eta_0 < \eta_1 < \ldots \quad c_0 > 0 \tag{33}$$

to check the geodesic incompleteness and the strength of non-BB singularities using geodesic equations and the definitions of Tipler (T) and Królak (K).

Classification of singularities.

Type	Name	t sing.	a(t_s)	$\varrho(t_s)$	p(t_s)	$\dot{p}(t_s)$ etc.	w(t_s)	T	K
0	Big-Bang (BB)	0	0	∞	∞	∞	finite	strong	strong
I	Big-Rip (BR)	t_s	∞	∞	∞	∞	finite	strong	strong
I_l	Little-Rip (LR)	∞	∞	∞	∞	∞	finite	strong	strong
I_p	Pseudo-Rip (PR)	∞	∞	finite	finite	finite	finite	weak	weak
II	Sudden Future (SFS)	t_s	a_s	ϱ_s	∞	∞	finite	weak	weak
II_g	Gen. Sudden Future (GSFS)	t_s	a_s	ϱ_s	p_s	∞	finite	weak	weak
III	Finite Scale Factor (FSF)	t_s	a_s	∞	∞	∞	finite	weak	strong
IV	Big-Separation (BS)	t_s	a_s	0	0	∞	∞	weak	weak
V	w-singularity (w)	t_s	a_s	0	0	0	∞	weak	weak

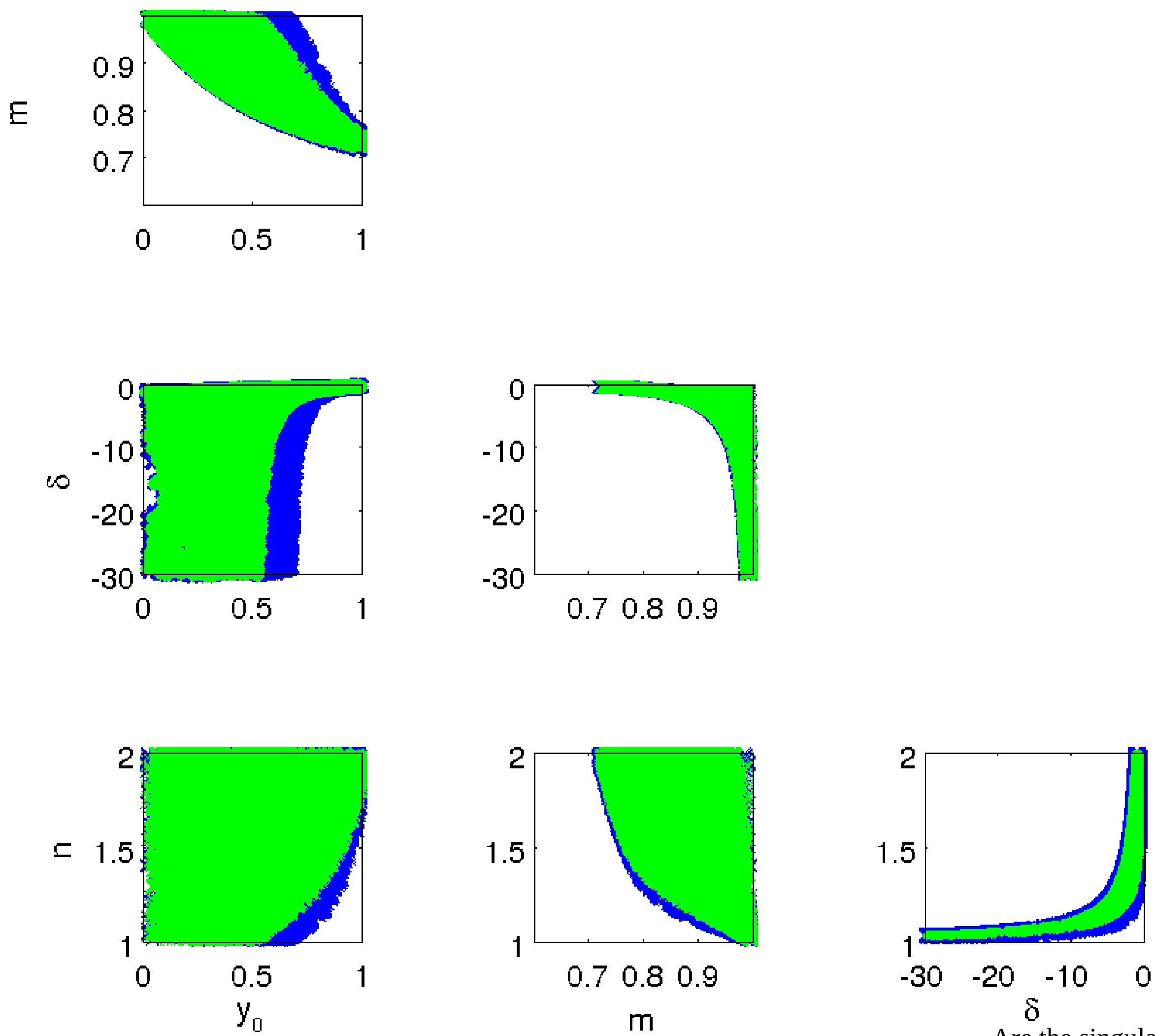
Non-BB singularities can mimic dark energy.



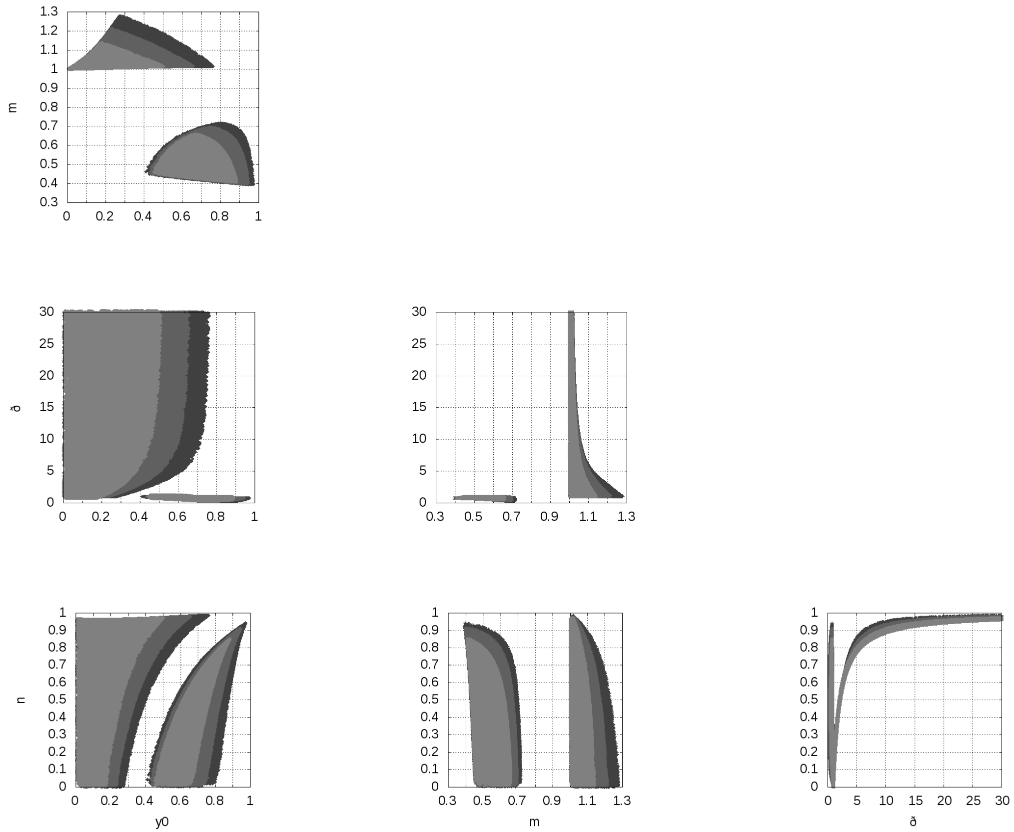
SFS - supernovae only (MPD et al. 2007): distance modulus $\mu_L = m - M$ for the CC model ($H_0 = 72 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$, $\Omega_{m0} = 0.26$, $\Omega_{\Lambda 0} = 0.74$) (dashed curve) and SFS model ($m = 2/3 = 0.6666, n = 1.9999, \delta = -0.471, y_0 = 0.99936$ - **SFS in 8.7 mln years**) (solid curve). Open circles - 'Gold' data; filled circles - SNLS data

SFS: supernovae, CMB, BAO (Denkiewicz et al. 2012)

Fits if $m \approx 0.72$, $w = -0.82$



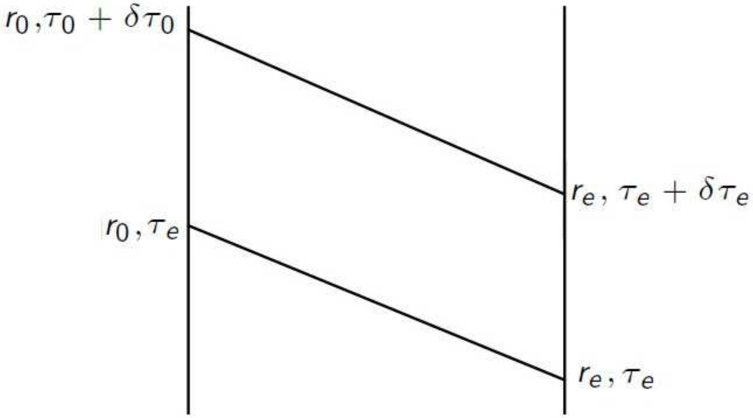
FSF: supernovae, CMB, BAO (Denkiewicz 2012)



$m = 2/3$ (dust) matter allowed in the past; FSF may happen in $2 \cdot 10^9$ years in future (stronger, and closer to big-bang since $a =$const, and big-bang has $a = 0$).

Redshift drift test.

Redshift drift (Sandage 1962) - the idea is to collect data from two light cones separated by 10-20 years to look for a change in redshift of a source as a function of time.



There is a relation between the times of emission of light by the source τ_e and $\tau_e + \Delta\tau_e$ and times of their observation at τ_o and $\tau_o + \Delta\tau_o$:

$$\int_{\tau_e}^{\tau_o} \frac{d\tau}{a(\tau)} = \int_{\tau_e+\Delta\tau_e}^{\tau_o+\Delta\tau_o} \frac{d\tau}{a(\tau)} \,, \tag{34}$$

which for small $\Delta\tau_e$ and $\Delta\tau_o$ reads as $\frac{\Delta\tau_e}{a(\tau_e)} = \frac{\Delta\tau_o}{a(\tau_o)}$.

Redshift drift test.

The redshift drift is defined as

$$\Delta z = z_e - z_0 = \frac{a(t_0 + \Delta t_0)}{a(t_e + \Delta t_e)} - \frac{a(t_0)}{a(t_e)} \ , \tag{35}$$

which can be expanded in series and to first order in Δt as

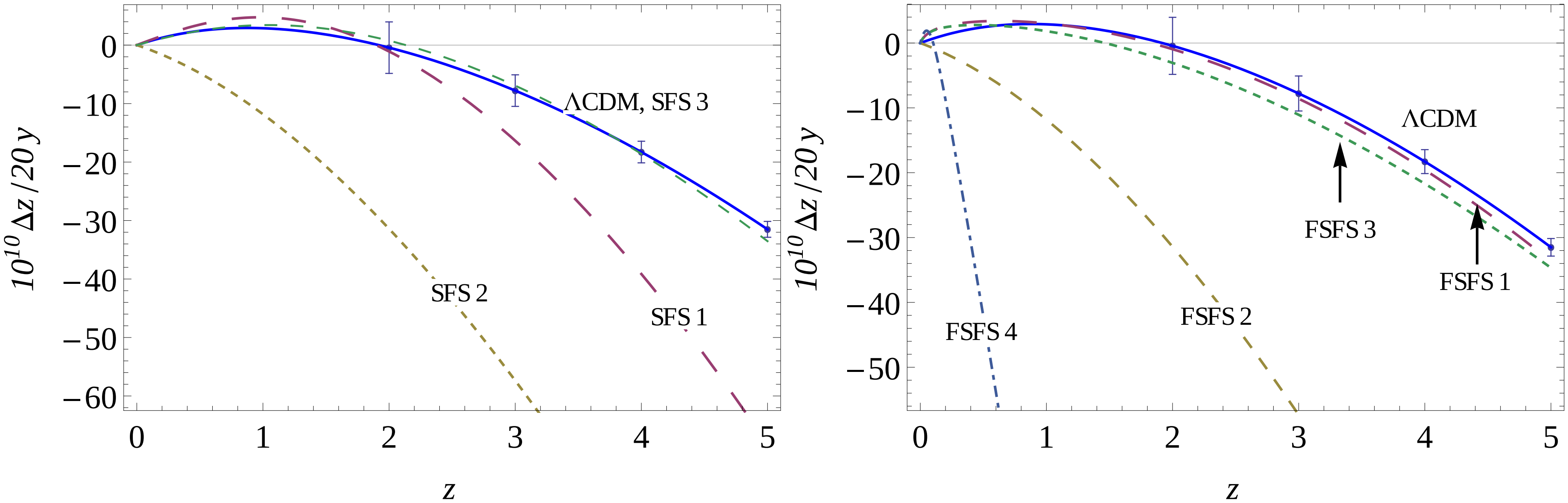
$$\begin{aligned} \Delta z &= \frac{a(t_0) + \dot{a}(t_0)\Delta t_0}{a(t_e) + \dot{a}(t_e)\Delta t_e} - \frac{a(t_0)}{a(t_e)} \\ &\approx \frac{a(t_0)}{a(t_e)} \left[\frac{\dot{a}(t_0)}{a(t_0)} \Delta t_0 - \frac{\dot{a}(t_e)}{a(t_e)} \Delta t_e \right] \ . \end{aligned} \tag{36}$$

Using above relations we have

$$\Delta z = \Delta t_0 \left[H_0 (1 + z) - H(t(z)) \right] = (1 + z) \frac{\Delta v}{c} \ , \tag{37}$$

where Δv is the velocity shift and $H(t(z))$ is given in a standard way.

Redshift drift for SFS, FSF (Denkiewicz, MPD, Martins, Vielzeuf, 2014).



SFS3, FSFS1, FSFS3 can mimic ΛCDM

SFS1, FSFS4 differ from ΛCDM significantly

SFS2, FSFS2 - dust Friedmann model

$H_0 = 67.3\ km/s/Mpc$ and $\Omega_{m0} = 0.315$ (Planck 2013).

redshift drift - parameters

	m	δ	n	t_0/t_s
SFS 1	$2/3$	-0.43	1.9999	0.99
SFS 2	$2/3$	0.0	1.9999	0.99
SFS 3	0.749	-0.45	1.99	0.77
FSFS 1	0.56	0.42	0.8	0.96
FSFS 2	$2/3$	0.0	0.7	0.79
FSFS 3	$2/3$	0.24	0.7	0.96
FSFS 4	1.15	7.5	0.81	0.51

RD planned to be measured by ELT-HIRES high-resolution ultra-stable spectrograph for the E-ELT (European Extermely Large Telescope) - Lyman-α forest. Also SKA (Square Kilometre Array), CHIME (The Canadian Hydrogen Intensity Mapping Experiment). Plus DECIGO/BBO - grav. wave related measurements.

4. The evolution through a singularity - averaging approach.

A.K. Raychaudhuri (PRL 80, 654 (1998)) proposed that one may average physical and kinematical scalars over the whole open spacetime (provided they vanish rapidly at spatial and temporal infinity) as follows

$$< \chi >= \lim_{x^a \to \infty} \frac{\int \int \int \int_{-x^a}^{x^a} \chi \sqrt{-g} d^4x}{\int \int \int \int_{-x^a}^{x^a} \sqrt{-g} d^4x} \tag{38}$$

By an open model it is meant that the ratio of the 3-volume hypersurfaces to a 4-volume of spacetime vanishes, i.e.,

$$\frac{\int \int \int \sqrt{|^3 g|} d^3x}{\int \int \int \int \sqrt{-g} d^4x} = 0. \tag{39}$$

His idea was to tight the vanishing of the average $< \chi >$ with the singularity avoidance in cosmology.

Spacetime averaging - density and pressure.

For the pressure, the energy density, and the average acceleration we have

$$< p >= - \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} \frac{\int_{t_0}^{t_1} a^3 \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt} \tag{40}$$

and

$$< \varrho >= \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} \frac{3 \int_{t_0}^{t_1} a^3 \left(\frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt}. \tag{41}$$

$$< \dot{\theta} >= \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} \frac{3 \int_{t_0}^{t_1} a^3 \left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt}. \tag{42}$$

SFS universe through an exotic singularity.

One is able to construct a hybrid model which allows Big-Bang, SFS, and finally Big-Crunch given by:

$$a_L(t) = a_s \left[\delta + \left(1 + \frac{t}{t_B}\right)^m (1-\delta) - \delta \left(-\frac{t}{t_B}\right)^n \right] \tag{43}$$

with $t_B < 0$ - the Big-Bang time, and $t = 0$ and SFS time;

$$a_R(t) = a_s \left[\delta + \left(1 - \frac{t}{t_C}\right)^m (1-\delta) - \delta \left(\frac{t}{t_C}\right)^n \right] \tag{44}$$

with $t_C > 0$ - the Big-Crunch time. In the high pressure regime $t \to 0$ these are approximated by

$$a_L \approx a_s \left[1 + \frac{m}{t_B} (1-\delta)\, t \right], \tag{45}$$

$$a_R \approx a_s \left[1 - \frac{m}{t_C} (1-\delta)\, t \right]. \tag{46}$$

Spacetime averaging - standard and phantom models

$$< p >_{stand} = \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} -\frac{4}{\gamma}\left(\frac{1}{\gamma} - 1\right)\frac{\int_{t_0}^{t_1} t^{2\left(\frac{1}{\gamma}-1\right)} dt}{\int_{t_0}^{t_1} t^{\frac{2}{\gamma}} dt} \to 0,$$

$$< \varrho >_{stand} = \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} -\frac{4}{3\gamma^2}\frac{\int_{t_0}^{t_1} t^{2\left(\frac{1}{\gamma}-1\right)} dt}{\int_{t_0}^{t_1} t^{\frac{2}{\gamma}} dt} \to 0$$

$$< p >_{ph} = \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} -\frac{4}{\mid \gamma \mid}\left(\frac{1}{\mid \gamma \mid} + 1\right)\frac{\int_{t_0}^{t_1} t^{-2\left(\frac{1}{|\gamma|}+1\right)} dt}{\int_{t_0}^{t_1} t^{-\frac{2}{|\gamma|}} dt} \to \infty,$$

$$< \varrho >_{ph} = \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} -\frac{4}{3\gamma^2}\frac{\int_{t_0}^{t_1} t^{-2\left(\frac{1}{|\gamma|}+1\right)} dt}{\int_{t_0}^{t_1} t^{-\frac{2}{|\gamma|}} dt} \to \infty$$

Spacetime averaging - SFS and FSF models

$$\dot{a}_L(t) = a_s \left[\frac{m}{t_B} \left(1 + \frac{t}{t_B}\right)^{m-1} (1-\delta) + \delta \frac{n}{t_B^n} (-t)^{n-1} \right] \tag{47}$$

$$\dot{a}_R(t) = a_s \left[-\frac{m}{t_C} \left(1 - \frac{t}{t_C}\right)^{m-1} (1-\delta) + \delta \frac{n}{t_C^n} (t)^{n-1} \right] \tag{48}$$

$$\frac{\ddot{a}_L}{a_s} = \frac{m(m-1)(1-\delta)}{t_B^2} \left(1 + \frac{t}{t_B}\right)^{m-2} - \frac{\delta n(n-1)}{t_B^n} (-t)^{n-2} \tag{49}$$

$$\frac{\ddot{a}_R}{a_s} = \frac{m(1-m)(1-\delta)}{t_C^2} \left(1 - \frac{t}{t_C}\right)^{m-2} + \frac{\delta n(n-1)}{t_C^n} t^{n-2} \tag{50}$$

Only the last terms blow up to give infinite pressure for $1 < n < 2$ at $t = 0$ so that we neglect other terms in a, $\dot{a}$ and $\ddot{a}$.

Spacetime averaging - SFS and FSF models

$$< \dot{\theta} >_{SFS,L} = \lim_{\substack{t_0 \to -t_B \\ t_1 \to 0}} -3n \frac{\int_{t_0}^{t_1} (-t)^{3n-2} dt}{\int_{t_0}^{t_1} (-t)^{3n} dt} \tag{51}$$

$$= \lim_{\substack{t_0 \to -t_B \\ t_1 \to 0}} -3n \frac{3n+1}{3n-1} \frac{(-t_1)^{3n-1} - (-t_0)^{3n-1}}{(-t_1)^{3n+1} - (-t_0)^{3n+1}} \to \frac{1}{t_B^2}$$

$$< \dot{\theta} >_{SFS,R} = \lim_{\substack{t_0 \to 0 \\ t_1 \to t_C}} 3n \frac{\int_{t_0}^{t_1} t^{3n-2} dt}{\int_{t_0}^{t_1} t^{3n} dt} \tag{52}$$

$$= \lim_{\substack{t_0 \to 0 \\ t_1 \to t_C}} 3n \frac{3n+1}{3n-1} \frac{t_1^{3n-1} - t_0^{3n-1}}{t_1^{3n+1} - t_0^{3n+1}} \to \frac{1}{t_C^2}$$

These averages are finite for SFS, but they may blow up for FSF if $0 < n < 1/3$!

Subtle differences between singularities.

- BB, BC singularities - all the energy conditions fulfilled, averages vanish (despite original claim of Raychaudhuri)
- BR singularity - no EC fulfilled, averages blow up
- SFS - only dominant energy violated, averages finite
- It seems that BR is stronger singularity that BB, BC on the ground of averaging.
- SFS is weaker, but FSF does not seem so.

This seems to be a new kind of a measure for the strength of singularities.

5. Varying constants removing or changing singularities.

- It has been shown that quantum effects (e.g. Houndjo 1008.0664; Houndjo et al. 1203.6084) may change the strength of exotic singularities (e.g. SFS can be changed into either FSF or BR or BC).
- On the other hand, varying constants cosmologies have been applied to solve standard cosmology problems such as the horizon and flatness problem (e.g. Albrecht, Magueijo 1999; Barrow 1999).
- **Here an idea** is to apply them to **remove of change the nature of singularities** in cosmology.

varying constants versus cosmic singularities.

Einstein-Friedmann equations generalize in varying speed of light (VSL) theories and varying gravitational constant G theories to (ϱ - mass density; $\varepsilon = \varrho c^2(t)$ - energy density in $Jm^{-3} = Nm^{-2} = kgm^{-1}s^{-2}$)

$$\varrho(t) = \frac{3}{8\pi G(t)}\left(\frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2}\right), \tag{53}$$

$$p(t) = -\frac{c^2(t)}{8\pi G(t)}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2}\right), \tag{54}$$

and the energy-momentum "conservation law" is

$$\dot{\varrho}(t) + 3\frac{\dot{a}}{a}\left(\varrho(t) + \frac{p(t)}{c^2(t)}\right) = -\varrho(t)\frac{\dot{G}(t)}{G(t)} + 3\frac{kc(t)\dot{c}(t)}{4\pi G a^2}. \tag{55}$$

General form of the scale factor.

We use a general form of the scale factor (MPD, K. Marosek, JCAP 02 (2013), 012), which **admits big-bang, big-rip, sudden future, finite scale factor and w-singularities** and reads as

$$a(t) = a_s \left(\frac{t}{t_s} \right)^m \exp \left(1 - \frac{t}{t_s} \right)^n , \tag{56}$$

with the constants t_s, a_s, m, n. For $k = 0$ we have

$$\varrho(t) = \frac{3}{8\pi G(t)} \left[\frac{m}{t} - \frac{n}{t_s} \left(1 - \frac{t}{t_s} \right)^{n-1} \right]^2 , \tag{57}$$

$$\begin{aligned} p(t) &= -\frac{c^2(t)}{8\pi G(t)} \left[\frac{m(3m-2)}{t^2} - 6\frac{mn}{tt_s} \left(1 - \frac{t}{t_s} \right)^{n-1} \right. \\ &+ \left. 3\frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s} \right)^{2(n-1)} + 2\frac{n(n-1)}{t_s^2} \left(1 - \frac{t}{t_s} \right)^{n-2} \right] . \end{aligned} \tag{58}$$

The scale factor - parametrization.

For $m < 0$ we have **a big-rip singularity** - $a \to \infty$, $\varrho \to \infty$, $p \to \infty$ at $t = 0$;
For $1 < n < 2$ we have **a sudden future singularity** (SFS) which appears at $t = t_s$ ($a = a_s$, $\varrho =$ const., $p \to \infty$);
For $0 < n < 1$ we have **a (stronger) finite scale factor singularity** (FSF) at $t = t_s$ ($a = a_s$, $\varrho \to \infty$, $p \to \infty$).

In fact, for $1 < n < 2$ only the last term in the pressure of the type $(1 - t/t_s)^{n-2}$ blows-up, while for $0 < n < 1$ two more terms $(1 - t/t_s)^{n-1}$ and $(1 - t/t_s)^{2(n-1)}$ do.

Removing singularities by varying constants

One bears in mind the scale factor (56), the energy density (57) and pressure (58)

Regularizing a Big-Bang singularity by varying G:

If

$$G(t) \propto \frac{1}{t^2} \tag{59}$$

which is a faster decrease than in Dirac's LNH $G \propto 1/t$, but influences less the temperature of the Earth constraint (Teller 1948).

Both divergence in ϱ and p are removed, though at the expense of having the "singularity" of strong gravitational coupling $G \to \infty$ at $t \to 0$.

In the Dirac's case, only the ϱ singularity can be removed.

removing singularities by varying constants: SFS

Regularizing an SFS singularity by varying c:

If

$$c(t) = c_0 \left(1 - \frac{t}{t_s}\right)^{\frac{p}{2}} , \tag{60}$$

then

$$p(t) = -\frac{c_0^2}{8\pi G} \left[\frac{m(3m-2)}{t^2} \left(1 - \frac{t}{t_s}\right)^p - 6\frac{mn}{tt_s} \left(1 - \frac{t}{t_s}\right)^{p+n-1} + 3\frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{p+2n-2} + 2\frac{n(n-1)}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{p+n-2} \right].$$

and the singularity of pressure is removed provided $p > 2 - n$, $(1 < n < 2)$.

removing singularities by varying constants: SFS.

Physical consequence: **light eventually stops** at the singularity. Same happens in loop quantum cosmology (LQC) where it is called the anti-newtonian limit $c = c_0\sqrt{1 - \varrho/\varrho_c} \to 0$ for $\varrho \to \varrho_c$ with ϱ_c being the critical density (Cailettau et al. 2012). The low-energy limit $\varrho \ll \varrho_0$ gives the standard limit $c \to c_0$.

It also appears naturally in Magueijo model ((Magueijo, PRD 63, 043502 (2001))) in which black holes are not reachable since the light stops at the horizon (despite they possess Schwarzschild singularity). An observer cannot reach this surface even in his finite proper time.

Strangely, both options $c = 0$ and $c = \infty$ are possible in Magueijo model.

removing singularities by varying constants: SFS

Removing an SFS singularity by varying G:

If we assume that

$$G(t) = G_0 \left(1 - \frac{t}{t_s}\right)^{-r} , \tag{61}$$

($r =$ const., $G_0 =$ const.) which changes (57) and (58) to

$$\begin{aligned} \varrho(t) &= \frac{3}{8\pi G_0}\left[\frac{m^2}{t^2}\left(1-\frac{t}{t_s}\right)^{r} - \frac{2mn}{tt_s}\left(1-\frac{t}{t_s}\right)^{r+n-1} \right.\\ &+ \left. \frac{n^2}{t_s^2}\left(1-\frac{t}{t_s}\right)^{r+2n-2}\right] , \end{aligned} \tag{62}$$

$$\begin{aligned} p(t) &= -\frac{c^2}{8\pi G_0}\left[\frac{m(3m-2)}{t^2}\left(1-\frac{t}{t_s}\right)^{r} - 6\frac{mn}{tt_s}\left(1-\frac{t}{t_s}\right)^{r+n-1} + 3\frac{n^2}{t_s^2}\left(1-\frac{t}{t_s}\right)^{r+2n-2} \right.\\ &+ \left. 2\frac{n(n-1)}{t_s^2}\left(1-\frac{t}{t_s}\right)^{r+n-2}\right] . \end{aligned} \tag{63}$$

removing or changing singularities by varying constants: SFS

From (62) and (63) it follows that an SFS singularity $(1 < n < 2)$ is regularized by varying gravitational constant when

$$r > 2 - n \ , \tag{64}$$

and an FSF singularity $(0 < 1 < n)$ is regularized when

$$r > 1 - n \ . \tag{65}$$

On the other hand, assuming that we have an SFS singularity and that

$$-1 < r < 0 \, , \tag{66}$$

we get that varying G **may change an SFS singularity onto a stronger FSF singularity** when

$$0 < r + n < 1 \, . \tag{67}$$

Subtleties:

- In order to regularize an SFS or an FSF singularity by varying $c(t)$, the **light should slow and eventually stop propagating** at a singularity. Similar effects were found in loop quantum cosmology (LQC) as well as in VSL for Schwarzschild horizon (Magueijo 2001) - speed of light is either zero or infinity at $r = r_s$. An observer cannot reach this surface even in his finite proper time.
- To regularize an SFS, FSF by varying gravitational constant $G(t)$ - **the strength of gravity has to become infinite** at a singularity. On the one hand, it is quite reasonable because of the requirement to **overcome an infinite (anti-)tidal forces** at the singularity, but on the other hand, it makes another singularity - **a singularity of strong coupling** for a physical field such as $G \propto 1/\Phi$. Such problems were already dealt with in superstring and brane cosmology where both the curvature singularity and a strong coupling singularity appeared (choice of coupling, quantum corrections).

6. Conclusions

- Currently one is able to **differentiate quite a number of cosmological singularities** with completely different properties - despite many of them are geodesically complete, they still lead to a **blow-up of physical quantities** such as scale factor, energy density, pressure, physical fields etc.
- **The exotic singularities can be influenced by varying constants**. It is possible to **remove or change** the type of these singularities with full physical consequences of this. However, we may face **new ”singularity” in a physical field** responsible for the variability of constants but this is what happens in physical theories (e.g. superstring) too.
- **Some singularities (Big-Bang, Big-Rip) are obviously the limits of cosmology (cannot be “gone through”) but some are not despite having admitting some ”pathologies” (infinities) which are not dangerous to the evolution of the universe.**

References

- A. Balcerzak and MPD, PRD **73**, 101301(R) (2006) (hep-th/0604034)
- MPD, T. Denkiewicz, M.A. Hendry, PRD **75**, 123524 (2007)
- MPD, T. Denkiewicz, PRD **79**, 063521 (2009)
- MPD, T. Denkiewicz, 0910.0023 AIP Conference Proceedings **1241**, 561 (2010).
- MPD, PLB 702, 320 (2011).
- H. Ghodsi, M. A. Hendry, MPD, T. Denkiewicz, Mon. Not. R. Astr. Soc. 414, 151725 (2011).
- T. Denkiewicz, MPD, H. Ghodsi, M. A. Hendry, Phys. Rev. D85, 083527 (2012).
- T. Denkiewicz, JCAP **07**, 036 (2012).
- MPD, K. Marosek, JCAP 02 (2013), 012 (arXiv: 1207.4038).
- T. Denkiewicz, MPD, C.J.A.P. Martins, P. Vielzeuf - 1402.0520, PRD to appear (2014) + in progress.