Exotic singularities in cosmology and varying constants

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Plan:

- 1. Introduction.
- 2. Standard and exotic singularities in cosmology.
- 3. The universe through an exotic singularity - averaging surprises.
- 4. Varying constants theories.
- 5. Varying constant versus cosmic singularities.
- 6. Conclusions.

1. Introduction.

Standard Einstein-Friedmann equations are two equations for three unknown functions of time $a(t), p(t), \varrho(t)$

$$\varrho = \frac{3}{8\pi G}\left(\frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\right), \qquad (1)$$

$$p = -\frac{1}{8\pi G}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\right). \qquad (2)$$

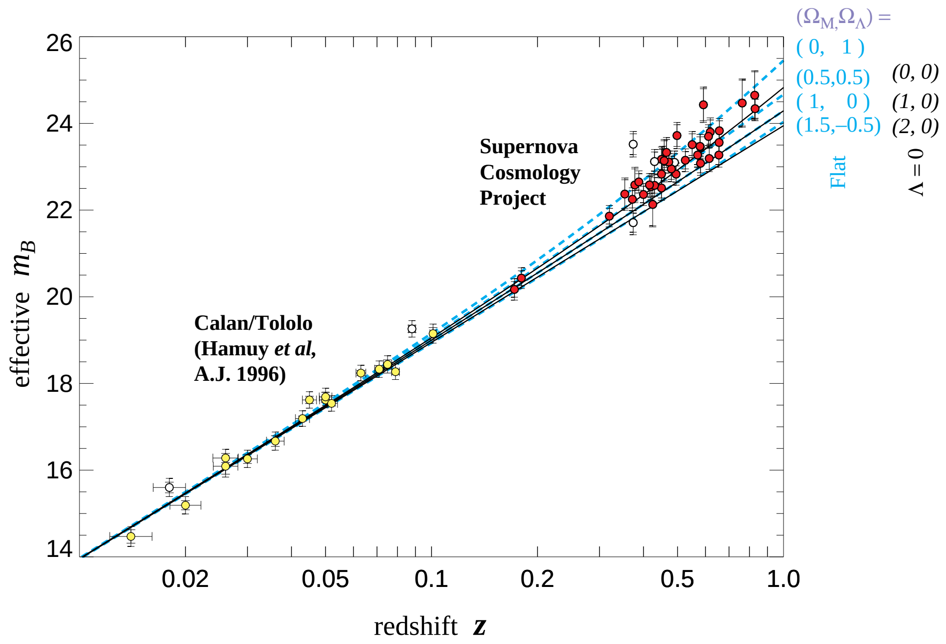
plus an equation of state, e.g., of a barotropic type ($w =$ const. ≥ -1):

$$p(t) = w\varrho(t) \qquad \to a(t) \propto t^{\frac{2}{3(w+1)}}. \qquad (3)$$

Until very recently (including first supernovae results) most of cosmologists studied only simplest - say **"standard" solutions - each of them starts with Big-Bang singularity** in which $a \to 0$, $\varrho, p \to \infty$
– one of them (of $K = +1$) terminates at the second singularity (Big-Crunch) where $a \to 0$, $\varrho, p \to \infty$
– the other two ($K = 0, -1$) continue to an asymptotic emptiness $\varrho, p \to 0$ for $a \to \infty$.

BB and BC exhibit geodesic incompleteness and curvature blow-up.

However, first supernovae observations ...



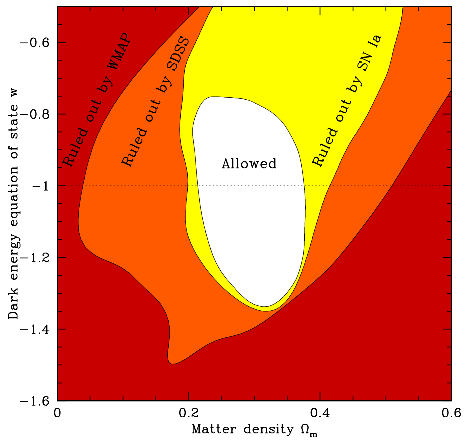
... gave evidence for the strong energy condition

$$\varrho + 3p \geq 0, \qquad \varrho + p \geq 0\ . \tag{4}$$

violation, but **the paradigm of the "standard" Big-Bang/Crunch singularities remained untouched.**

2. Standard and exotic singularities in cosmology.

WMAP + SDSS + Supernovae **combined bound on the dark energy barotropic index** w (Tegmark et al. (2004)); recent: Shafer & Huterer 1312.1668



- **showed that there was no sharp cut-off of the data at $p = -\varrho$!!!** so that
- **the dark energy with $p < -\varrho$ (phantom) could be admitted!**
- **cosmic "no-hair" theorem violation - even a small fraction of phantom dark energy may dominate the evolution - Big-Rip singularity**
- NEC, WEC, DEC violated!

Big-Rip (type I) as an exotic (neither BB nor BC) singularity.

Since for phantom $w < -1$, then for convenience we may take

$$| w + 1 | = -(w + 1) > 0 \,, \tag{5}$$

so $a(t) = t^{-2/3|w+1|}$ and the conservation law for phantom gives

$$\varrho \propto a^{3|w+1|} \,. \tag{6}$$

- Conclusion: the bigger the universe grows, the denser it is, and **it becomes dominated by phantom (which overcomes Λ-term)** – an exotic future singularity appears – Big-Rip $\varrho, p \to \infty$ for $a \to \infty$
- Curvature invariants R^2, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ diverge at Big-Rip
- In a Big-Rip scenario everything is pulled apart on the approach to a Big-Rip in a reverse order (Caldwell et al. PRL ’03). Specifically, for $w = -3/2$ Big-Rip will happen in 20 Gyr.

Sudden Future Singularity (type II) as an exotic singularity.

Observational support for a Big-Rip gave a push to studies some other exotic types of singularities as possible sources of dark energy

Barrow (2004) dropped an assumption about the imposition of the equation of state (3)

$$p \neq p(\varrho), \tag{7}$$

and investigated how the energy density and pressure evolves if one assumes the analytic form of the scale factor only:

$$a(t) = a_s \left[\delta + (1-\delta)\, y^m - \delta\, (1-y)^n\right] , \qquad y \equiv \frac{t}{t_s} \tag{8}$$

where $a_s \equiv a(t_s) = \text{const.}$ and $\delta, m, n = \text{const.}$

$$\dot{a} = a_s \left[\frac{m}{t_s}\,(1-\delta)\, y^{m-1} + \delta \frac{n}{t_s}\,(1-y)^{n-1}\right] , \tag{9}$$

$$\ddot{a} = \frac{a_s}{t_s^2} \left[m\,(m-1)\,(1-\delta)\, y^{m-2} - \delta n\,(n-1)\,(1-y)^{n-2}\right] . \tag{10}$$

Sudden Future Singularity ...

Provided

$$1 < n < 2, \tag{11}$$

one gets apart from a Big-Bang at $t = 0$ there is a new type of singularity at $t = t_s$ - a Sudden Future Singularity (SFS) (or type II - Nojiri, Odintsov, Tsujikawa 2005) which:

– manifests as a singularity of pressure p (or $\ddot{a}$) only

– leads to the dominant energy condition violation only. In fact we have:

$$\begin{aligned} a = \text{const.}, \quad \dot{a} = \text{const.} \quad \varrho = \text{const.} \\ \ddot{a} \to -\infty \quad p \to \infty \quad \text{for} \quad t \to t_s \end{aligned} \tag{12}$$

Interesting point:

Schwarzschild horizon at $r = r_g$ - metric singular, curvature invariants regular,

Sudden Future Singularity at $t = t_s$ - metric regular, curvature invariants diverge.

commonly appear in LQC (Cailletau et al. PRL 101, 251302 (2008)) on the contrary to big-bang and big-rip which are avoided in LQC (e.g. Sami et al. gr-qc/0605113).
This is due to quadratic term in the Friedmann equation (e.g. Bojowald PRL '02, gr-qc/0601085) which simulates negative brane tension (extra timelike dimensions - e.g. Shtanov and Sahni PLB, 557 (2003), 1):

$$H^2 = \frac{1}{3m_{pl}^2}\left(\rho - \frac{\rho^2}{\rho_c}\right) - \frac{k}{a^2} , \tag{13}$$

where the critical density is

$$\rho_c \equiv \frac{\sqrt{3}}{16\pi\gamma^3 G^2 \hbar^2} , \tag{14}$$

and γ is the Barbero-Immirzi parameter ($\gamma \approx 0.2375$, see Meissner gr-qc/0407052).

Generalized Sudden Future singularities (type IIg).

Sudden future singularities may be generalized to GSFS if we take a general scale factor time derivative of an order r:

$$a^{(r)} = a_s \left[\frac{m(m-1)...(m-r+1)}{t_s^r} (1-\delta)\, y^{m-r} + (-1)^{r-1} \delta \frac{n(n-1)...(n-r+1)}{t_s^r} (1-y)^{n-r} \right], \qquad (15)$$

and choose (Barrow 2004, Lake 2004) $r-1<n<r$. Then for any integer r we have a singularity in the scale factor derivative $a^{(r)}$, and consequently in the appropriate pressure derivative $p^{(r-2)}$.

None of the energy conditions (EC) are violated for $r \geq 3$!!!

Finite Scale Factor (type III) and Big Separation (type IV).

The new exotic singularities were found as Type III singularities which we will call Finite Scale Factor - FSF singularities are characterized by the following conditions (Nojiri, Odintsov, Tsujikawa 2005):

$$a = a_s = \text{const.}, \varrho, \dot{a}_s \to \infty, |p|, \ddot{a}_s \to \infty$$

The simplest way to get them is to apply the scale factor as given previously for SFS, i.e.,

$$a(t) = a_s \left[\delta + (1-\delta)\, y^m - \delta\, (1-y)^n\right] , \qquad y \equiv \frac{t}{t_s} \tag{16}$$

where $a_f \equiv a(t_f) = \text{const.}$ and $\delta, A, m, n = \text{const.}$, but with the range of parameter n changed from $1 < n < 2$ onto

$$0 < n < 1$$

Big Separation - BS (type IV)

Type IV singularity is when (Nojiri, Odintsov, Tsujikawa 2005):

$$a = a_s = \text{const.}, \varrho \to 0, p \to 0, \dddot{a}, \ddot{H} \to \infty \text{ etc.}$$

and so it is similar to Generalized Sudden Future singularity with only one exception: it also gives the divergence of the barotropic index in the barotropic equation of state

$$p(t) = w(t)\varrho(t)$$

i.e.,

$$w(t) \to \infty$$

Barotropic index $w-$singularity (Type V)

Assuming the following type of scale factor (MPD, Denkiewicz 2009):

$$\begin{aligned} a(t) &= \frac{a_s}{1-\frac{3\gamma}{2}\left(\frac{n-1}{n-\frac{2}{3\gamma}}\right)^{n-1}} \\ &+ \frac{1-\frac{2}{3\gamma}}{n-\frac{2}{3\gamma}} \frac{n a_s}{1-\frac{2}{3\gamma}\left(\frac{n-\frac{2}{3\gamma}}{n-1}\right)^{n-1}} \left(\frac{t}{t_s}\right)^{\frac{2}{3\gamma}} \\ &+ \frac{a_s}{\frac{3\gamma}{2}\left(\frac{n-1}{n-\frac{2}{3\gamma}}\right)^{n-1}-1} \left(1-\frac{1-\frac{2}{3\gamma}}{n-\frac{2}{3\gamma}}\frac{t}{t_s}\right)^{n}, \end{aligned} \tag{17}$$

with the admissible values of the parameters: $\gamma = w + 1 > 0$ and $n \neq 1$.

$w-$singularity

one gets a blow-up of the effective barotropic index, i.e.,

$$w(t_s) = \frac{1}{3}\left[2q(t_s) - 1\right] \to \infty \,, \tag{18}$$

accompanied by

$$p(t_s) \to 0; \quad \varrho(t_s) \to 0 \,. \tag{19}$$

There is a kind of duality between the Big-Bang and the w-singularity in the form

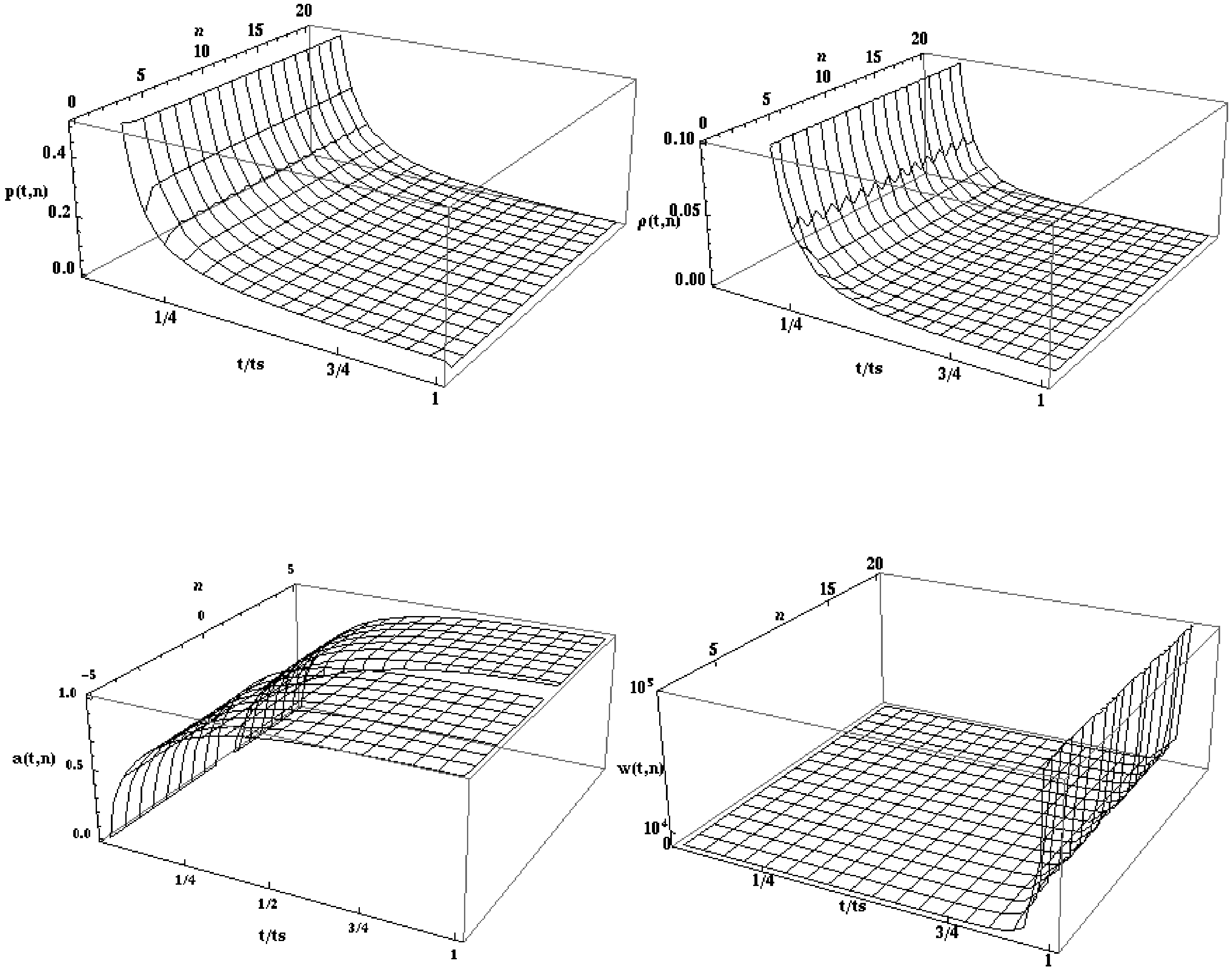
$$p_{BB} \leftrightarrow \frac{1}{p_w}, \quad \varrho_{BB} \leftrightarrow \frac{1}{\varrho_w}, \quad w_{BB} \leftrightarrow \frac{1}{w_w} \quad . \tag{20}$$

In other words:

$$p_{BB} \to \infty; \, \varrho_{BB} \to \infty; \, w_{BB} \to 0; \, a_{BB} \to 0$$
$$p_w \to 0; \, \varrho_w \to 0; \, w_w \to \infty; \, a_w \to a_s = \text{const.}$$

w-sing. appear in: $f(R)$ gravity (Starobinsky '80), scalar field gravity (Setare, Saridakis '09), and brane gravity (Sahni, Shtanov '05)

$w-$duality



Classification of exotic sing. (Nojiri et al. 2005, MPD & Denkiewicz 2010).

- Type 0 - Big-Bang (Big-Crunch) $a \to 0$, $p \to \infty$, $\varrho \to \infty$
- Type I - Big-Rip $a(t_s) \to \infty$ ($t_s < \infty$), $p \to \infty$, $\varrho \to \infty$ (Caldwell 2002)
- Type II - Sudden Future (includes Big Boost and Big-Brake) $a(t_s) =$ const., $\varrho =$ const., $p \to \infty$ (Barrow 2004)
- Type IIg - Generalized Sudden Future $a(t_s)$= const., $\varrho =$ const., $p =$const., $\dddot{a} \to \infty$ etc., $w < \infty$ (Barrow 2004)
- Type III - Finite Scale Factor (also Big-Freeze) $a(t_s)$ = const., $\varrho \to \infty$, $p \to \infty$ (NOT 2005, Denkiewicz 2011)
- Type IV - Big Separation: $a(t_s)$= const., $p = \varrho = 0$, $w \to \infty$, $\dddot{a} \to \infty$ etc. (NOT 2005) (and generalizations $p = \varrho =$const. Yurov 2010)
- Type V - w-singularity $a(t_s)$= const., $p = \varrho = 0$, $w \to \infty$ (MPD, Denkiewicz 2009) (and generalizations $p =$const. Yurov 2010)
- Little-Rip $a(t_s) \to \infty$, $\varrho(t_s) \to \infty$ ($t_s \to \infty$),
- Pseudo-Rip $\varrho(t_s) < \infty$ ($t_s \to \infty$) (Frampton et al. 2011, 2012)

Are these really singularities - strength?

As an example let us take an SFS which is determined by a blow-up of the Riemann tensor and its derivatives.

Geodesics do not feel SFSs at all, since geodesic equations are not singular for $a_s = a(t_s) =$ const. (Fernandez-Jambrina, Lazkoz PRD 74, 064030 (2006))

$$\left(\frac{dt}{d\tau}\right)^2 = A + \frac{P^2 + KL^2}{a^2(t)} , \tag{21}$$

$$\frac{dr}{d\tau} = \frac{P_1 cos\phi + P_2 \sin\phi}{a^2(t)} \sqrt{1 - Kr^2} , \tag{22}$$

$$\frac{d\phi}{d\tau} = \frac{L}{a^2(t) r^2} . \tag{23}$$

Geodesic deviation equation

$$\frac{D^2 n^\alpha}{d\lambda^2} + R^\alpha_{\ \beta\gamma\delta} u^\beta n^\gamma u^\delta = 0 , \tag{24}$$

feels SFS since at $t = t_s$ we have the Riemann tensor $R^\alpha_{\ \beta\gamma\delta} \to \infty$.

Classification of exotic singularities - strength.

- Tipler's (Phys. Lett. A64, 8 (1977)) definition (of a strong singularity):
  $\int_0^{\tau} d\tau' \int_0^{\tau'} d\tau'' |R^i_{ajb} u^a u^b|$
  diverges on the approach to a singularity at $\tau = \tau_s$
- i.e. an extended object is crushed to zero volume (represented by three linearly independent, vorticity-free geodesic deviation vectors at p along causal geodesic l) at the singularity by infinite tidal forces
- Królak's (CQG 3, 267 (1988)) definition (of a strong singularity):
  $\int_0^{\tau} d\tau' |R^i_{ajb} u^a u^b|$
  diverges on the approach to a singularity at $\tau = \tau_s$
- i.e. the expansion of every future-directed congruence of null (timelike) geodesics emanating from point p and containig l becomes negative somewhere on l

Classification of exotic singularities - strength.

Fernandez-Jambrina (PRD 82, 124004 (2010)) used Puiseux series expansion

$$a(t) = c_0 + (t_s - t)^{\eta_0} + c_1 (t_s - t)^{\eta_1} + c_2 (t_s - t)^{\eta_2} + \dots \qquad \eta_0 < \eta_1 < \dots \quad c_0 > 0 \tag{25}$$

to check the strength of exotic singularities (T - Tipler; K - Królak)

Balcerzak and MPD PRD'06) considered classical Polyakov strings

$$S = -\frac{T}{2} \int d\tau d\sigma \eta^{ab} g_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \tag{26}$$

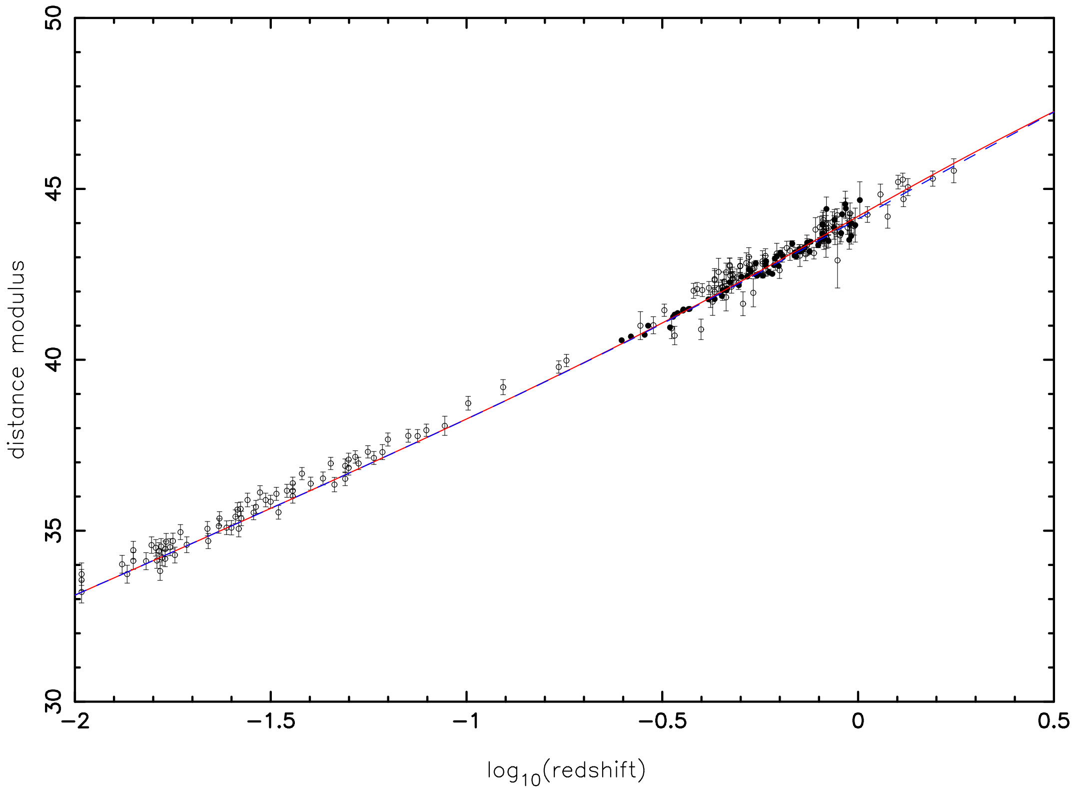
with an invariant size $S(\tau) = 2\pi a(\eta(\tau)) R(\tau)$ (circular ansatz with radius R) falling into exotic singularities to show that they are: infinitely stretched $S \to \infty$ at Big-Rip while for SFS the scale factor is finite at η-time at SFS so that the invariant string size is also finite. The same is true for type III, IV and generalized SFS.

This means strings are not destroyed at weak singularities.

Classification of singularities in FRW cosmology.

Type	Name	t sing.	a(t_s)	$\varrho(t_s)$	p(t_s)	$\dot{p}(t_s)$ etc.	w(t_s)	T	K
0	Big-Bang (BB)	0	0	∞	∞	∞	finite	strong	strong
I	Big-Rip (BR)	t_s	∞	∞	∞	∞	finite	strong	strong
I_l	Little-Rip (LR)	∞	∞	∞	∞	∞	finite	strong	strong
I_p	Pseudo-Rip (PR)	∞	∞	finite	finite	finite	finite	weak	weak
II	Sudden Future (SFS)	t_s	a_s	ϱ_s	∞	∞	finite	weak	weak
II_g	Gen. Sudden Future (GSFS)	t_s	a_s	ϱ_s	p_s	∞	finite	weak	weak
III	Finite Scale Factor (FSF)	t_s	a_s	∞	∞	∞	finite	weak	strong
IV	Big-Separation (BS)	t_s	a_s	0	0	∞	∞	weak	weak
V	w-singularity (w)	t_s	a_s	0	0	0	∞	weak	weak

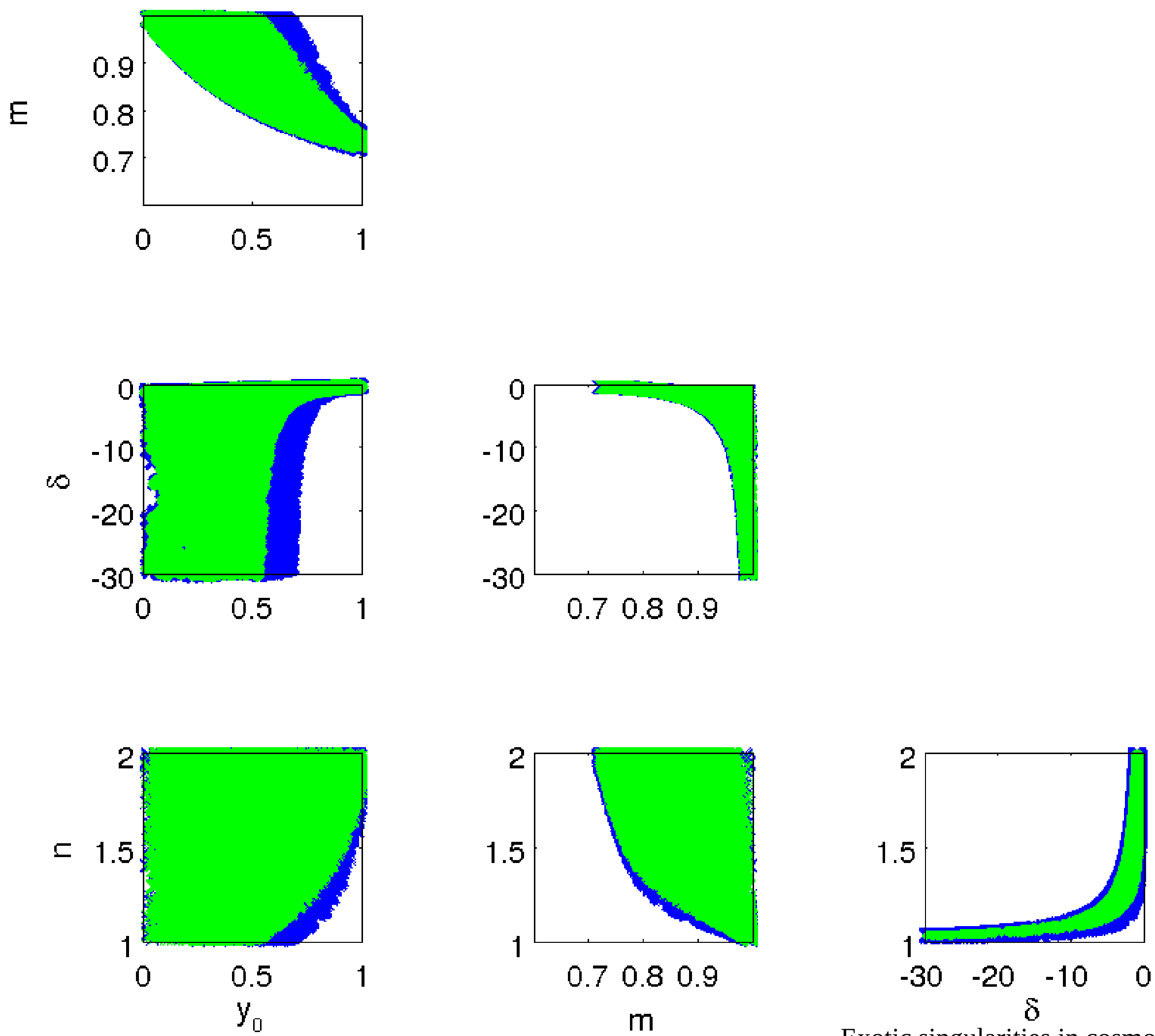
Exotic singularities can mimic dark energy.



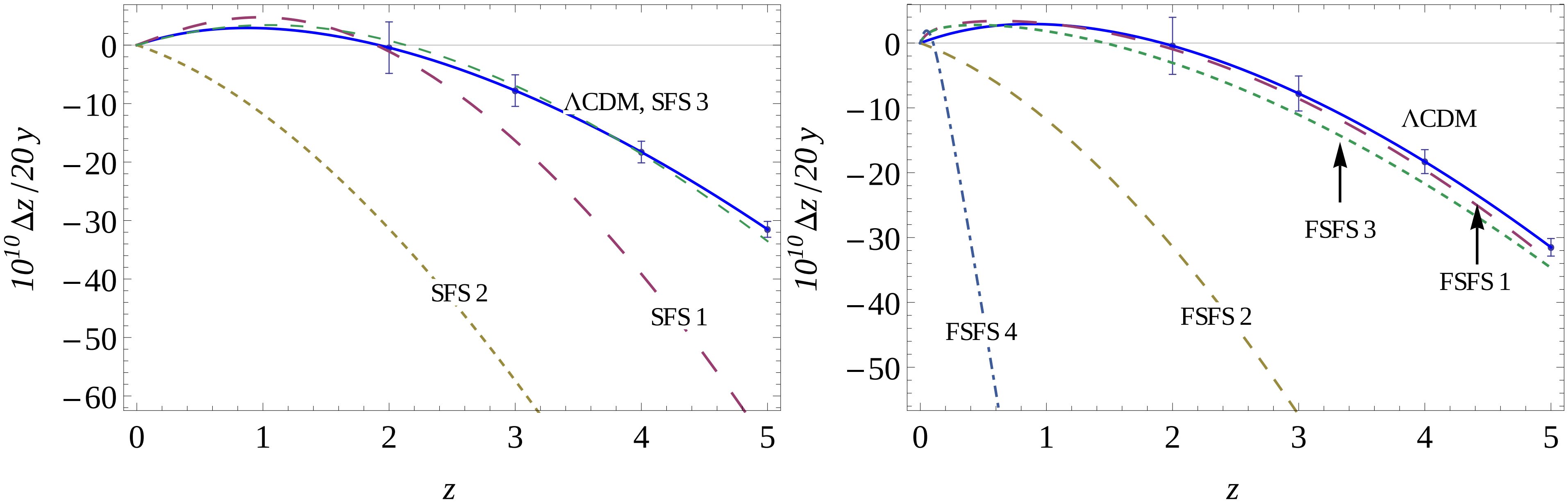
SFS - supernovae only (MPD et al. 2007): distance modulus $\mu_L = m - M$ for the CC model ($H_0 = 72 \text{kms}^{-1}\text{Mpc}^{-1}$, $\Omega_{m0} = 0.26$, $\Omega_{\Lambda 0} = 0.74$) (dashed curve) and SFS model ($m = 2/3 = 0.6666, n = 1.9999, \delta = -0.471, y_0 = 0.99936$) (solid curve). Open circles are for the 'Gold' data and filled circles are for SNLS data.

SFS, FSFS: supernovae, CMB, BAO (Denkiewicz et al. 2012)

Fits if $m \approx 0.72$, $w = -0.82$



Another test: redshift drift (Denkiewicz, MPD, Martins, Vielzeuf, 2014).



redshift drift - parameters

	m	δ	n	t_0/t_s
SFS 1	$2/3$	-0.43	1.9999	0.99
SFS 2	$2/3$	0.0	1.9999	0.99
SFS 3	0.749	-0.45	1.99	0.77
FSFS 1	0.56	0.42	0.8	0.96
FSFS 2	$2/3$	0.0	0.7	0.79
FSFS 3	$2/3$	0.24	0.7	0.96
FSFS 4	1.15	7.5	0.81	0.51

$H_0 = 67.3\ km/s/Mpc$ and $\Omega_{m0} = 0.315$ (cf. Planck data 2013).

3. The universe through a singularity - averaging surprises.

A.K. Raychaudhuri (PRL 80, 654 (1998)) proposed that one may average physical and kinematical scalars over the whole open spacetime provided they vanish rapidly at spatial and temporal infinity as follows

$$< \chi >= \lim_{x^a \to \infty} \frac{\int\int\int\int_{-x^a}^{x^a} \chi \sqrt{-g} d^4x}{\int\int\int\int_{-x^a}^{x^a} \sqrt{-g} d^4x} \tag{27}$$

By an open model it is meant that the ratio of the 3-volume hypersurfaces to a 4-volume of spacetime vanishes, i.e.,

$$\frac{\int\int\int \sqrt{|^3 g|} d^3x}{\int\int\int\int \sqrt{-g} d^4x} = 0. \tag{28}$$

His idea was to tight the vanishing of the average $< \chi >$ with the singularity avoidance in cosmology.

Spacetime averaging - density and pressure.

For the pressure, the energy density, and the average acceleration we have

$$< p >= - \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} \frac{\int_{t_0}^{t_1} a^3 \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt} \tag{29}$$

and

$$< \varrho >= \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} \frac{3 \int_{t_0}^{t_1} a^3 \left(\frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt}. \tag{30}$$

$$< \dot{\theta} >= \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} \frac{3 \int_{t_0}^{t_1} a^3 \left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt}. \tag{31}$$

SFS universe through an exotic singularity.

One is able to construct a hybrid model which allows Big-Bang, Sudden Future Singularity and finally Big-Crunch given by:

$$a_L(t) = a_s \left[\delta + \left(1 + \frac{t}{t_B} \right)^m (1-\delta) - \delta \left(-\frac{t}{t_B} \right)^n \right] \quad (32)$$

with $t_B < 0$ - the Big-Bang time, and $t = 0$ and SFS time;

$$a_R(t) = a_s \left[\delta + \left(1 - \frac{t}{t_C} \right)^m (1-\delta) - \delta \left(\frac{t}{t_C} \right)^n \right] \quad (33)$$

with $t_C > 0$ - the Big-Crunch time. In the high pressure regime $t \to 0$ these are approximated by

$$a_L \approx a_s \left[1 + \frac{m}{t_B} (1-\delta)\, t \right], \quad (34)$$

$$a_R \approx a_s \left[1 - \frac{m}{t_C} (1-\delta)\, t \right]. \quad (35)$$

Spacetime averaging - standard and phantom models

$$< p >_{stand} = \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} -\frac{4}{\gamma}\left(\frac{1}{\gamma} - 1\right) \frac{\int_{t_0}^{t_1} t^{2\left(\frac{1}{\gamma}-1\right)} dt}{\int_{t_0}^{t_1} t^{\frac{2}{\gamma}} dt} \to 0,$$

$$< \varrho >_{stand} = \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} -\frac{4}{3\gamma^2} \frac{\int_{t_0}^{t_1} t^{2\left(\frac{1}{\gamma}-1\right)} dt}{\int_{t_0}^{t_1} t^{\frac{2}{\gamma}} dt} \to 0$$

$$< p >_{ph} = \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} -\frac{4}{\mid \gamma \mid}\left(\frac{1}{\mid \gamma \mid} + 1\right) \frac{\int_{t_0}^{t_1} t^{-2\left(\frac{1}{|\gamma|}+1\right)} dt}{\int_{t_0}^{t_1} t^{-\frac{2}{|\gamma|}} dt} \to \infty,$$

$$< \varrho >_{ph} = \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} -\frac{4}{3\gamma^2} \frac{\int_{t_0}^{t_1} t^{-2\left(\frac{1}{|\gamma|}+1\right)} dt}{\int_{t_0}^{t_1} t^{-\frac{2}{|\gamma|}} dt} \to \infty$$

Spacetime averaging - SFS and FSF models

$$\dot{a}_L(t) = a_s \left[\frac{m}{t_B} \left(1 + \frac{t}{t_B} \right)^{m-1} (1-\delta) + \delta \frac{n}{t_B^n} (-t)^{n-1} \right] \quad (36)$$

$$\dot{a}_R(t) = a_s \left[-\frac{m}{t_C} \left(1 - \frac{t}{t_C} \right)^{m-1} (1-\delta) + \delta \frac{n}{t_C^n} (t)^{n-1} \right] \quad (37)$$

$$\frac{\ddot{a}_L}{a_s} = \frac{m(m-1)(1-\delta)}{t_B^2} \left(1 + \frac{t}{t_B} \right)^{m-2} - \frac{\delta n(n-1)}{t_B^n} (-t)^{n-2} \quad (38)$$

$$\frac{\ddot{a}_R}{a_s} = \frac{m(1-m)(1-\delta)}{t_C^2} \left(1 - \frac{t}{t_C} \right)^{m-2} + \frac{\delta n(n-1)}{t_C^n} t^{n-2} \quad (39)$$

Only the last terms blow up to give infinite pressure for $1 < n < 2$ at $t = 0$ so that we neglect other terms in a, $\dot{a}$ and $\ddot{a}$.

Spacetime averaging - SFS and FSF models

$$< \dot{\theta} >_{SFS,L} = \lim_{\substack{t_0 \to -t_B \\ t_1 \to 0}} -3n \frac{\int_{t_0}^{t_1} (-t)^{3n-2} dt}{\int_{t_0}^{t_1} (-t)^{3n} dt} \tag{40}$$

$$= \lim_{\substack{t_0 \to -t_B \\ t_1 \to 0}} -3n \frac{3n+1}{3n-1} \frac{(-t_1)^{3n-1} - (-t_0)^{3n-1}}{(-t_1)^{3n+1} - (-t_0)^{3n+1}} \to \frac{1}{t_B^2}$$

$$< \dot{\theta} >_{SFS,R} = \lim_{\substack{t_0 \to 0 \\ t_1 \to t_C}} 3n \frac{\int_{t_0}^{t_1} t^{3n-2} dt}{\int_{t_0}^{t_1} t^{3n} dt} \tag{41}$$

$$= \lim_{\substack{t_0 \to 0 \\ t_1 \to t_C}} 3n \frac{3n+1}{3n-1} \frac{t_1^{3n-1} - t_0^{3n-1}}{t_1^{3n+1} - t_0^{3n+1}} \to \frac{1}{t_C^2}$$

These averages are finite for SFS, but they may blow up for FSF if $0 < n < 1/3$!

Subtle differences between singularities.

- BB, BC singularities - all the energy conditions fulfilled, averages vanish (despite original claim of Raychaudhuri)
- BR singularity - no EC fulfilled, averages blow up
- SFS - only dominant energy violated, averages finite
- It seems that BR is stronger singularity that BB, BC on the ground of averaging.
- SFS is weaker, but FSF does not seem so.

This seems to be a new kind of a measure for the strength of singularities.

4. Varying constants theories.

- It has been shown that quantum effects (e.g. Houndjo et al.: 1203.6084) may change the strength of exotic singularities (e.g. SFS to FSF).
- On the other hand, varying constants cosmologies have been applied to solve standard cosmology problems such as the horizon and flatness problem (e.g. Albrecht, Magueijo 1999).
- **Our idea** is to apply them to **solve the singularity problem** in cosmology.
- We can also ask if varying constants theories **can soften/strengthen** the standard and exotic singularities?

varying constants theories

First fully quantitative framework: Brans-Dicke scalar-tensor gravity (1961)

The gravitational constant G is associated with an average gravitational potential (scalar field) ϕ surrounding a given particle:

$< \phi >= GM/(c/H_0) \propto 1/G = 1.35 \times 10^{28} g/cm$. The scalar field gives the strength of gravity

$$G = \frac{1}{16\pi\Phi} \tag{42}$$

With the action

$$S = \int d^4x\sqrt{-g}\left(\Phi R - \frac{\omega}{\Phi}\partial_\mu\Phi\partial^\mu\Phi + \Lambda + L_m\right) \tag{43}$$

it relates to low-energy-effective superstring theory for $\omega = -1$

String coupling constant (running) $g_s = \exp{(\phi/2)}$ changes in time with ϕ - the dilaton and $\Phi = \exp{(-\phi)}$.

varying constants theories

Varying speed of light theories (VSL): Albrecht & Magueijo model (AM model) (1999)(Barrow 1999; Magueijo 2003):

$$c^4 = \psi(x^\mu) \tag{44}$$

and so the action is

$$S = \int d^4x \sqrt{-g} \left[\frac{\psi(R + 2\Lambda)}{16\pi G} + L_m + L_\psi \right] \tag{45}$$

AM model breaks Lorentz invariance (relativity principle and light principle) - preferred frame (cosmological or CMB) in which the field is minimally coupled to gravity.

Solves basic problems of standard cosmology: horizon problem and flatness problem.

Ansatz: Friedmann with $\rho = \rho_0 a^{-3\gamma}$, $c(t) = c_0 a^n$ - solution if $n \leq (1/2)(2 - 3\gamma)$.

varying constants theories

Magueijo covariant (conformally) and locally invariant model (2000, 2001):

$$\psi = \ln\left(\frac{c}{c_0}\right) \quad \text{or} \quad c = c_0 e^{\psi} , \tag{46}$$

with the action

$$S = \int d^4x \sqrt{-g} \left[\frac{c_0^4 e^{\alpha\psi} (R + 2\Lambda + L_\psi)}{16\pi G} + e^{\beta\psi} L_m \right] , \tag{47}$$

with

$$L_\psi = \kappa(\psi) \nabla_\mu \psi \nabla^\mu \psi . \tag{48}$$

Further assumption: $\alpha - \beta = 4$.

Interesting subcases:

$\alpha = 4; \beta = 0$ - Brans-Dicke with $\phi_{BD} = e^{4\psi}/G$ and $\kappa(\psi) = 16\omega_{BD}(\phi_{BD})$.

$\alpha = 0; \beta = -4$ - minimal VSL theory.

varying constants theories

Varying fine structure constant α (or charge $e = e_0\epsilon(x^\mu)$ theories (Webb et al. 1999, Sandvik 2002)

$$S = \int d^4x\sqrt{-g}\left(\psi R - \frac{\omega}{2}\partial_\mu\psi\partial^\mu\psi - \frac{1}{4}f_{\mu\nu}f^{\mu\nu}e^{-2\psi} + L_m\right) \tag{49}$$

with $\psi = \ln\epsilon$ and $f_{\mu\nu} = \epsilon F_{\mu\nu}$.
Assume linear expansion $e^\psi = 1 - 8\pi G\zeta(\psi - \psi_0) = 1 - \Delta\alpha/\alpha$ with the constraint on the local equivalence principle violence $\mid \zeta \mid \leq 10^{-3}$. The relation to dark energy is:

$$\gamma = w + 1 = \frac{(8\pi G\frac{d\psi}{d\ln a})^2}{\Omega_\psi} \ . \tag{50}$$

While mimicking the dark energy this can be tested by spectrograph CODEX (COsmic Dynamics EXplorer) attached to the planned E-ELT (European Extremely Large Telescope) which measures the redshift drift (or Sandage-Loeb effect) for $2 < z < 5$ (Vielzeuf and Martins 2012).

Some observational constraints:

- $|(dG/dt)/G| < 9 \cdot 10^{-13}/year$ - from primordial nucleosynthesis (Accetta et al. 1990);
- $|(dG/dt)/G| < 1.6 \cdot 10^{-12}/year$ - from helioseismology (Guenther et al. 1998);
- $|(dG/dt)/G| < (4 \pm 9) \cdot 10^{-13}/year$ - from lunar laser ranging (LLR) (Williams et al. 1996);
- $\Delta\alpha/\alpha = (3.85 \pm 5.65) \cdot 10^{-8}$ - from Oklo phenomenon (Shlyakhter 1976, Petrov et al. 2006);
- $\Delta\alpha/\alpha = (-8 \pm 16) \cdot 10^{-7}$ - from meteorite dating (long-lived beta decays) (Olive et al. 2003);
- $\Delta\alpha/\alpha = (-0.5 \pm 1.3) \cdot 10^{-5}$ - from quasar absorption spectra with redshifts $2.33 < z < 3.08$ (Murphy et al. 2001);
- $\Delta\beta/\beta = (5.7 \pm 3.8) \cdot 10^{-5}$ ($\beta = m_e/m_p$) - from quasar absorption spectra (Ivanchik et al. 2005).

5. Varying constants versus cosmic singularities.

We consider the Friedmann universes in varying speed of light (VSL) theories and varying gravitational constant G theories as follows (ϱ - mass density; $\varepsilon = \varrho c^2(t)$ - energy density in $Jm^{-3} = Nm^{-2} = kgm^{-1}s^{-2}$)

$$\varrho(t) = \frac{3}{8\pi G(t)}\left(\frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2}\right) , \tag{51}$$

$$p(t) = -\frac{c^2(t)}{8\pi G(t)}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2}\right) , \tag{52}$$

and the energy-momentum “conservation law” is

$$\dot{\varrho}(t) + 3\frac{\dot{a}}{a}\left(\varrho(t) + \frac{p(t)}{c^2(t)}\right) = -\varrho(t)\frac{\dot{G}(t)}{G(t)} + 3\frac{kc(t)\dot{c}(t)}{4\pi G a^2} . \tag{53}$$

General form of the scale factor.

We use a general form of the scale factor (MPD, K. Marosek, JCAP 02 (2013), 012), which **admits big-bang, big-rip, sudden future, finite scale factor and w-singularities** and reads as

$$a(t) = a_s \left(\frac{t}{t_s}\right)^m \exp\left(1 - \frac{t}{t_s}\right)^n , \tag{54}$$

with the constants t_s, a_s, m, n. For $k = 0$ we have

$$\varrho(t) = \frac{3}{8\pi G(t)} \left[\frac{m}{t} - \frac{n}{t_s}\left(1 - \frac{t}{t_s}\right)^{n-1}\right]^2 , \tag{55}$$

$$\begin{aligned} p(t) = & -\frac{c^2(t)}{8\pi G(t)} \left[\frac{m(3m-2)}{t^2} - 6\frac{mn}{tt_s}\left(1 - \frac{t}{t_s}\right)^{n-1} \right. \\ & + \left. 3\frac{n^2}{t_s^2}\left(1 - \frac{t}{t_s}\right)^{2(n-1)} + 2\frac{n(n-1)}{t_s^2}\left(1 - \frac{t}{t_s}\right)^{n-2}\right] . \end{aligned} \tag{56}$$

The scale factor - parametrization.

For $m < 0$ we have **a big-rip singularity** - $a \to \infty$, $\varrho \to \infty$, $p \to \infty$ at $t = 0$;
For $1 < n < 2$ we have **a sudden future singularity** (SFS) which appears at $t = t_s$ ($a = a_s$, $\varrho =$ const., $p \to \infty$);
For $0 < n < 1$ we have **a stronger finite scale factor singularity** (FSF) at $t = t_s$ ($a = a_s$, $\varrho \to \infty$, $p \to \infty$).

In fact, for $1 < n < 2$ only the last term in the pressure of the type $(1 - t/t_s)^{n-2}$ blows-up, while for $0 < n < 1$ two more terms $(1 - t/t_s)^{n-1}$ and $(1 - t/t_s)^{2(n-1)}$ do.

Regularizing singularities by varying constants

One bears in mind the scale factor (54), the energy density (55) and pressure (56)

Regularizing a Big-Bang singularity by varying G:

If

$$G(t) \propto \frac{1}{t^2} \tag{57}$$

which is a faster decrease than in Dirac's LNH $G \propto 1/t$, but influences less the temperature of the Earth constraint (Teller 1948).

Both divergence in ϱ and p are removed, though at the expense of having the "singularity" of strong gravitational coupling $G \to \infty$ at $t \to 0$.

In the Dirac's case, only the ϱ singularity can be removed.

regularizing singularities by varying constants: SFS

Regularizing an SFS singularity by varying c:

If

$$c(t) = c_0 \left(1 - \frac{t}{t_s}\right)^{\frac{p}{2}} , \tag{58}$$

then

$$p(t) = -\frac{c_0^2}{8\pi G} \left[\frac{m(3m-2)}{t^2} \left(1 - \frac{t}{t_s}\right)^{p} \right.$$
$$\left. - \quad 6\frac{mn}{tt_s} \left(1 - \frac{t}{t_s}\right)^{p+n-1} + 3\frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{p+2n-2} + 2\frac{n(n-1)}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{p+n-2} \right] .$$

and the singularity of pressure is regularized provided $p > 2 - n, (1 < n < 2)$.

regularizing singularities by varying constants: SFS.

Physical consequence: **light eventually stops** at the singularity. Same happens in loop quantum cosmology (LQC) where it is called the anti-newtonian limit $c = c_0\sqrt{1 - \varrho/\varrho_c} \to 0$ for $\varrho \to \varrho_c$ with ϱ_c being the critical density (Cailettau et al. 2012). The low-energy limit $\varrho \ll \varrho_0$ gives the standard limit $c \to c_0$.

It also appears naturally in Magueijo model ((Magueijo, PRD 63, 043502 (2001))) in which black holes are not reachable since the light stops at the horizon (despite they possess Schwarzschild singularity).

Strangely, both options $c = 0$ and $c = \infty$ are possible in Magueijo model.

regularizing singularities by varying constants: w-sing.

In the limit $m \to 0$ we have an exotic singularity scale factor given by $a(t) = a_s \exp\left(1 - t/t_s\right)$ and so from (55) and (56) we have

$$\varrho_{ex}(t) = \frac{3}{8\pi G(t)} \frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{2(n-1)} , \tag{59}$$

$$p_{ex}(t) = -\frac{c^2(t)}{8\pi G(t)} \left[3\frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{2(n-1)} + 2\frac{n(n-1)}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{n-2}\right] \tag{60}$$

so that

$$w_{ex}(t) = \frac{p_{ex}(t)}{\varepsilon_{ex}(t)} = -\left[1 + \frac{2}{3}\frac{n-1}{n}\frac{1}{\left(1 - \frac{t}{t_s}\right)^n}\right] = -\left[\frac{1}{3} - \frac{2}{3}q_{ex}(t)\right] , \tag{61}$$

which is a w-singularity for $n > 2$ ($p = \varrho = 0$, $w_{ex} \to \infty$). Its regularization by varying $c(t)$ is impossible since there is no c-dependence here.

regularizing singularities by varying constants: SFS

Regularizing an SFS singularity by varying G:

If we assume that

$$G(t) = G_0 \left(1 - \frac{t}{t_s}\right)^{-r} , \tag{62}$$

($r =$ const., $G_0 =$ const.) which changes (55) and (56) to

$$\begin{aligned} \varrho(t) &= \frac{3}{8\pi G_0}\left[\frac{m^2}{t^2}\left(1-\frac{t}{t_s}\right)^{r} - \frac{2mn}{tt_s}\left(1-\frac{t}{t_s}\right)^{r+n-1} \right. \\ &+ \left. \frac{n^2}{t_s^2}\left(1-\frac{t}{t_s}\right)^{r+2n-2}\right] , \end{aligned} \tag{63}$$

$$\begin{aligned} p(t) &= -\frac{c^2}{8\pi G_0}\left[\frac{m(3m-2)}{t^2}\left(1-\frac{t}{t_s}\right)^{r} - 6\frac{mn}{tt_s}\left(1-\frac{t}{t_s}\right)^{r+n-1} + 3\frac{n^2}{t_s^2}\left(1-\frac{t}{t_s}\right)^{r+2n-2} \right. \\ &+ \left. 2\frac{n(n-1)}{t_s^2}\left(1-\frac{t}{t_s}\right)^{r+n-2}\right] . \end{aligned} \tag{64}$$

regularizing singularities by varying constants: SFS

From (63) and (64) it follows that an SFS singularity $(1 < n < 2)$ is regularized by varying gravitational constant when

$$r > 2 - n \ , \tag{65}$$

and an FSF singularity $(0 < 1 < n)$ is regularized when

$$r > 1 - n \ . \tag{66}$$

On the other hand, assuming that we have an SFS singularity and that

$$-1 < r < 0 \, , \tag{67}$$

we get that varying G **may change an SFS singularity onto a stronger FSF singularity** when

$$0 < r + n < 1 \, . \tag{68}$$

Regularizing singularities: (anti-)Chaplygin gas

The equation of state of the (anti-)Chaplygin gas reads as

$$p(t) = \pm \frac{A}{\varepsilon(t)} = \pm \frac{A}{\varrho(t)c^2(t)} \qquad (A > 0) \ , \tag{69}$$

where the "-" sign is for Chaplygin gas while the "+" sign is for anti-Chaplygin gas case and the unit of A is the energy density(=pressure) square $J^2 m^{-6}$.
Inserting (69) into (53) gives

$$\dot{\varrho}(t) + 3\frac{\dot{a}}{a}\left(\frac{\varrho^2 c^4(t) \mp A}{\varrho(t)c^4(t)}\right) = -\varrho(t)\frac{\dot{G}(t)}{G(t)} + 3\frac{kc(t)\dot{c}(t)}{4\pi G(t)a^2} \ . \tag{70}$$

We assume both varying $G = G(t)$ and $c = c(t)$ though with zero curvature $(k = 0)$ as follows

$$\varrho(t)c^2(t) = B = \text{const.} \ , \tag{71}$$

regularizing singularities: (anti-)Chaplygin gas

The solution of (70) reads as

$$\varrho(t)a^{3\gamma}(t)G(t) = E = \text{const.}\ , \tag{72}$$

where we have defined

$$\gamma \equiv \frac{B^2 \mp A}{B^2} \tag{73}$$

Putting the standard big-bang scale factor $a(t) = (t/t_s)^{2/3\gamma}$, we finally have

$$\varrho(t) = \frac{Et_s^2}{t^2 G(t)}\ , \qquad p(t) = \mp\frac{A}{B} = \text{const.}\ , \tag{74}$$

which give $\varrho \to \infty$ and $p(0) = 0$ provided $G(0) = \text{const.} \neq 0$. The singularity at $t = 0$ in ϱ and p **can be regularized** by taking $G(t) \propto 1/t^2$ at the expense of having a constant pressure (cosmological term) instead of zero pressure.

Subtleties:

- In order to regularize an SFS or an FSF singularity by varying $c(t)$, the **light should slow and eventually stop propagating** at a singularity. Similar effects were found in loop quantum cosmology (LQC) as well as in VSL for Schwarzschild horizon (Magueijo 2001) - speed of light is either zero or infinity at $r = r_s$. An observer cannot reach this surface even in his finite proper time.
- To regularize an SFS, FSF by varying gravitational constant $G(t)$ - **the strength of gravity has to become infinite** at a singularity. On the one hand, it is quite reasonable because of the requirement to **overcome an infinite (anti-)tidal forces** at the singularity, but on the other hand, it makes another singularity - **a singularity of strong coupling** for a physical field such as $G \propto 1/\Phi$. Such problems were already dealt with in superstring and brane cosmology where both the curvature singularity and a strong coupling singularity appeared (choice of coupling, quantum corrections).

6. Conclusions

- Currently one is able to **differentiate quite a number of cosmological singularities** with completely different properties - despite many of them are geodesically complete, they still lead to a **blow-up of physical quantities** such as scale factor, energy density, pressure, physical fields etc.
- Some of these singularities **may serve as dark energy**, especially if they are quite close in the near future. For example, an SFS may even appear in 8.7 Myr with no contradiction with bare supernovae data. It **can be fitted** to a combined SnIa, CMB, BAO, and redshift drift data for specific choice of the parameters.
- Our proposal is to investigate **how the singularities are influenced by varying physical constants**. In particular, we may look for the answer if it is possible to **"regularize" (remove infinities) or change** these singularities and what are the physical consequences of this, because what we face is usually the **new "singularity" in a physical fields** which act to remove/change a particular type of a singularity.