
The zoo of exotic singularities in cosmology

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Note:

Any similarity to

zooniverse.org

project

is purely coincidental!

References

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Plan:

- 1. Introduction.
- 2. Exotic singularities in cosmology.
- 3. Classification and observational verification.
- 4. Averaging approach - the universe through an exotic singularity.
- 5. Varying constants versus cosmic singularities.
- 6. Conclusions.

1. Introduction.

Standard Einstein-Friedmann equations are **two equations for three unknown functions** of time $a(t), p(t), \rho(t)$

$$\rho = \frac{3}{8\pi G} \left(\frac{\dot{a}^2}{a^2} + \frac{K}{a^2} \right), \quad (1)$$

$$p = -\frac{1}{8\pi G} \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{K}{a^2} \right). \quad (2)$$

To solve needs **an equation of state**, e.g., of a barotropic type ($w = \text{const.} \geq -1$):

$$p(t) = w\rho(t) \quad \rightarrow \quad a(t) \propto t^{\frac{2}{3(w+1)}}. \quad (3)$$

Until very recently (including first supernovae results) mainly the simplest - say **“standard” solutions**

- **were used each of them starts with Big-Bang singularity** in which $a \rightarrow 0, \rho, p \rightarrow \infty$

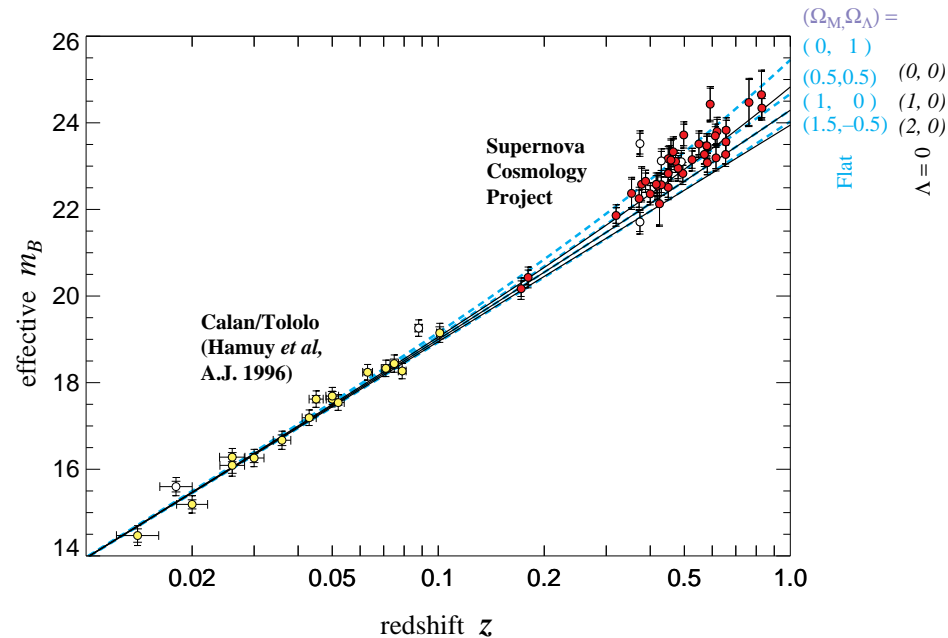
- one of them (of $K = +1$) terminates at the second singularity (**Big-Crunch**) where $a \rightarrow 0,$

$\rho, p \rightarrow \infty$

- the other two ($K = 0, -1$) continue to an **asymptotic emptiness** $\rho, p \rightarrow 0$ for $a \rightarrow \infty$.

BB and BC exhibit **geodesic incompleteness** and **curvature blow-up**.

However, first supernovae observations ...



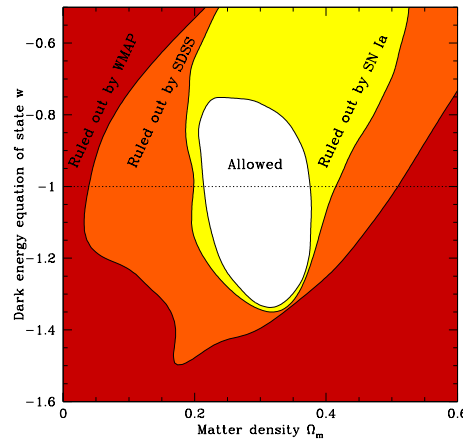
... gave evidence for the **strong** energy condition

$$\rho + 3p \geq 0, \quad \rho + p \geq 0. \quad (4)$$

violation, but **the paradigm of the “standard” Big-Bang/Crunch singularities remained untouched.**

2. Exotic singularities in cosmology.

WMAP + SDSS + Supernovae **combined bound on the dark energy barotropic index** w (Tegmark et al. (2004)); recent: Shafer & Huterer 1312.1668



- the dark energy with $p < -\rho$ (phantom) could be admitted!
- cosmic “no-hair” theorem violation - even a small fraction of phantom dark energy may dominate the evolution \rightarrow Big-Rip singularity
- N(ull) E(nergy) C(ondition) $\rho + p \geq 0$, W(eak) E(nergy) C(ondition) $\rho + p \geq 0, \rho \geq 0$, D(ominant) E(nergy) C(ondition) $|p| \leq \rho, \rho \geq 0$ are violated!!!

Big-Rip (BR - type I) as an exotic (neither BB nor BC) singularity.

Since for phantom $w < -1$, then for convenience we may take

$$|w + 1| = -(w + 1) > 0, \quad (5)$$

so $a(t) = t^{-2/3|w+1|}$ and the conservation law for phantom gives

$$\rho \propto a^{3|w+1|}. \quad (6)$$

- Conclusion: **the bigger** the universe grows, **the denser** it is, and **it becomes dominated by phantom (which overcomes Λ -term) – an exotic future singularity appears – Big-Rip** $\rho, p \rightarrow \infty$ for $a \rightarrow \infty$
- Curvature invariants $R^2, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ **diverge** at Big-Rip
- In a Big-Rip scenario everything is **pulled apart** on the approach to a Big-Rip in a reverse order (Caldwell et al. PRL '03). Specifically, for $w = -3/2$ Big-Rip will happen in 20 Gyr.

Sudden Future Singularity (SFS - type II) as an exotic singularity.

Surprise of a Big-Rip gave a push to studies some other exotic types of singularities as possible sources of dark energy

Barrow (2004) (based on earlier study Barrow, Galloway, Tipler 1986) **dropped an assumption about the imposition of the equation of state (3)**

$$p \neq p(\rho), \quad (7)$$

but assumed an analytic form of the scale factor instead:

$$a(t) = a_s [\delta + (1 - \delta) y^m - \delta (1 - y)^n] , \quad y \equiv \frac{t}{t_s} \quad (8)$$

where $a_s \equiv a(t_s) = \text{const.}$ and $\delta, m, n = \text{const.}$

$$\dot{a} = a_s \left[\frac{m}{t_s} (1 - \delta) y^{m-1} + \delta \frac{n}{t_s} (1 - y)^{n-1} \right] , \quad (9)$$

$$\ddot{a} = \frac{a_s}{t_s^2} \left[m(m-1)(1-\delta)y^{m-2} - \delta n(n-1)(1-y)^{n-2} \right] . \quad (10)$$

Sudden Future Singularity ...

Provided

$$1 < n < 2, \quad (11)$$

one gets a Big-Bang at $t = 0$ as well as a new type of singularity at $t = t_s$ - a **Sudden Future Singularity (SFS)** (or type II - Nojiri, Odintsov, Tsujikawa 2005) which:

- manifests as a singularity of pressure p (or \ddot{a}) only
- leads to the **dominant energy condition violation only**. In fact we have:

$$\begin{aligned} a = \text{const.}, \quad \dot{a} = \text{const.}, \quad \rho = \text{const.} \\ \ddot{a} \rightarrow -\infty \quad p \rightarrow \infty \quad \text{for} \quad t \rightarrow t_s \end{aligned} \quad (12)$$

Interesting observations:

Schwarzschild horizon at $r = r_g$ - **metric singular**, **curvature invariants regular**,

Sudden Future Singularity at $t = t_s$ - **metric regular**, **curvature invariants**

diverge.

Sudden Future Singularities ...

commonly appear in LQC (Cailletau et al. PRL 101, 251302 (2008)) on the contrary to big-bang and big-rip which are avoided in LQC (e.g. Sami et al. gr-qc/0605113).

This is due to quadratic term in the Friedmann equation (e.g. Bojowald PRL '02, gr-qc/0601085) which simulates negative brane tension (extra timelike dimensions - e.g. Shtanov and Sahni PLB, 557 (2003), 1):

$$H^2 = \frac{1}{3m_{pl}^2} \left(\rho - \frac{\rho^2}{\rho_c} \right) - \frac{k}{a^2}, \quad (13)$$

where the critical density is

$$\rho_c \equiv \frac{\sqrt{3}}{16\pi\gamma^3 G^2 \hbar^2}, \quad (14)$$

and γ is the Barbero-Immirzi parameter ($\gamma \approx 0.2375$, Meissner gr-qc/0407052).

Generalized Sudden Future singularities (type IIg).

Sudden Future Singularities may be generalized to GSFS if we take a general scale factor time derivative of an order r :

$$a^{(r)} = a_s \left[\frac{m(m-1)\dots(m-r+1)}{t_s^r} (1-\delta) y^{m-r} + (-1)^{r-1} \delta \frac{n(n-1)\dots(n-r+1)}{t_s^r} (1-y)^{n-r} \right], \quad (15)$$

and choose (Barrow 2004, Lake 2004) $r-1 < n < r$. Then for any integer r we have a **singularity** in the scale factor derivative $a^{(r)}$, and consequently **in** the appropriate **pressure derivative** $p^{(r-2)}$.

None of the energy conditions (EC) is violated for $r \geq 3!!!$

Finite Scale Factor (FSF - type III).

The new exotic singularities were found as Type III singularities which we will call **Finite Scale Factor - FSF** singularities are characterized by the following conditions (Nojiri, Odintsov, Tsujikawa 2005):

$$a = a_s = \text{const.}, \rho, \dot{a}_s \rightarrow \infty, |p|, \ddot{a}_s \rightarrow \infty$$

The simplest way to get them is to apply the scale factor as given previously for SFS, i.e.,

$$a(t) = a_s [\delta + (1 - \delta) y^m - \delta (1 - y)^n] , \quad y \equiv \frac{t}{t_s} \quad (16)$$

where $a_f \equiv a(t_f) = \text{const.}$ and $\delta, A, m, n = \text{const.}$, but with the range of parameter n changed from $1 < n < 2$ onto

$$0 < n < 1$$

Big Separation - BS (type IV)

Type IV singularity is when (Nojiri, Odintsov, Tsujikawa 2005):

$$a = a_s = \text{const.}, \rho \rightarrow 0, p \rightarrow 0, \dot{p}, \ddot{a}, \ddot{H} \rightarrow \infty \text{ etc.}$$

and so it is **similar** to Generalized Sudden Future singularity with only **one exception**: it also gives the divergence of the barotropic index in the barotropic equation of state

$$p(t) = w(t)\rho(t)$$

i.e.,

$$w(t) \rightarrow \infty$$

Barotropic index w –singularity (Type V)

Assuming the following type of scale factor (MPD, Denkiewicz 2009):

$$\begin{aligned} a(t) &= \frac{a_s}{1 - \frac{3\gamma}{2} \left(\frac{n-1}{n-\frac{2}{3\gamma}} \right)^{n-1}} \\ &+ \frac{1 - \frac{2}{3\gamma}}{n - \frac{2}{3\gamma}} \frac{na_s}{1 - \frac{2}{3\gamma} \left(\frac{n-\frac{2}{3\gamma}}{n-1} \right)^{n-1}} \left(\frac{t}{t_s} \right)^{\frac{2}{3\gamma}} \\ &+ \frac{a_s}{\frac{3\gamma}{2} \left(\frac{n-1}{n-\frac{2}{3\gamma}} \right)^{n-1} - 1} \left(1 - \frac{1 - \frac{2}{3\gamma}}{n - \frac{2}{3\gamma}} \frac{t}{t_s} \right)^n, \end{aligned} \quad (17)$$

with the admissible values of the parameters: $\gamma = w + 1 > 0$ and $n \neq 1$.

w-singularity

one gets a blow-up of the effective barotropic index, i.e.,

$$w(t_s) = \frac{1}{3} [2q(t_s) - 1] \rightarrow \infty, \quad (18)$$

accompanied by

$$p(t_s) \rightarrow 0; \quad \varrho(t_s) \rightarrow 0. \quad (19)$$

There is a kind of **duality between the Big-Bang and the *w*-singularity** in the form

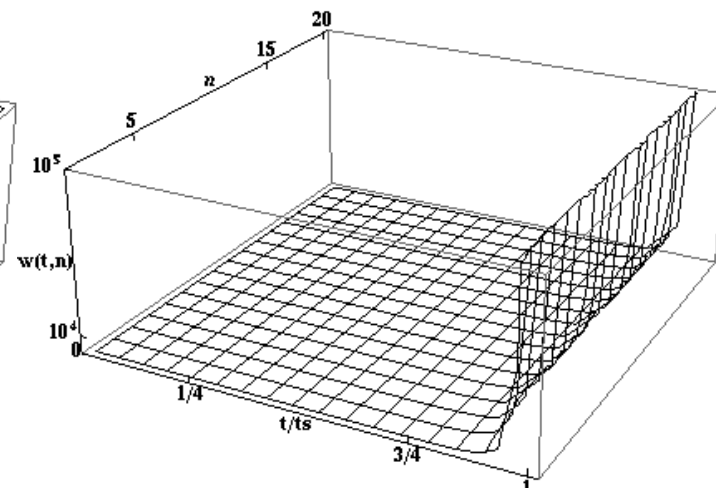
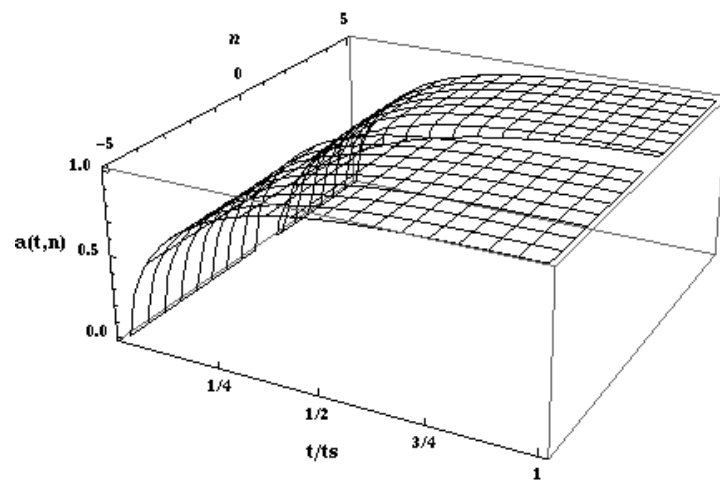
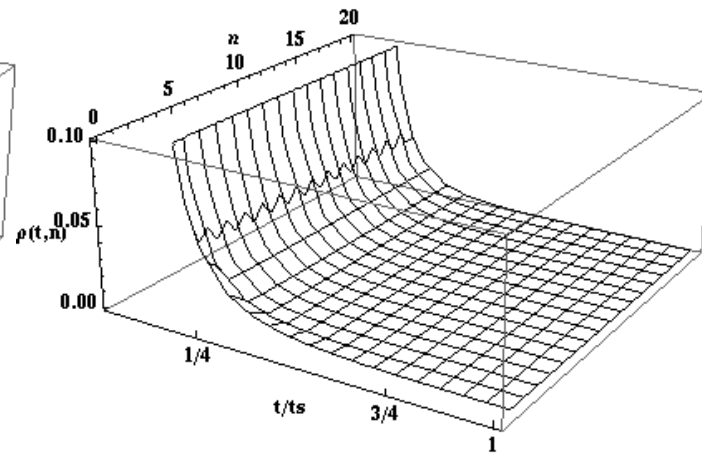
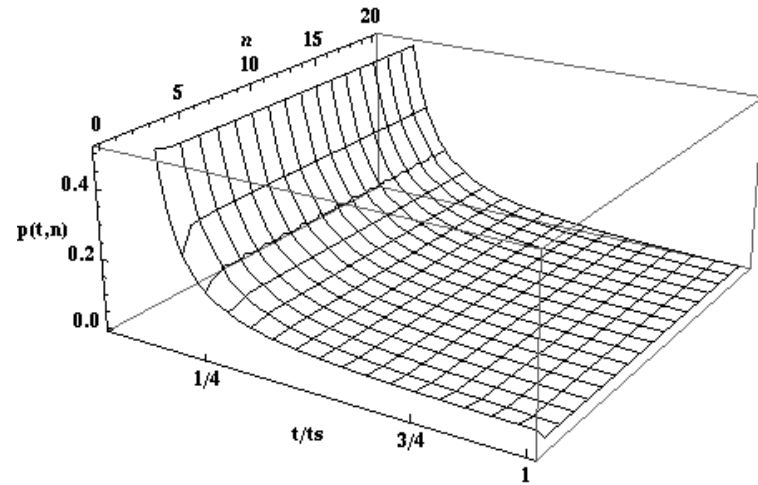
$$p_{BB} \leftrightarrow \frac{1}{p_w}, \quad \varrho_{BB} \leftrightarrow \frac{1}{\varrho_w}, \quad w_{BB} \leftrightarrow \frac{1}{w_w}. \quad (20)$$

In other words:

$$\begin{aligned} p_{BB} &\rightarrow \infty; \varrho_{BB} \rightarrow \infty; w_{BB} \rightarrow 0; a_{BB} \rightarrow 0 \\ p_w &\rightarrow 0; \varrho_w \rightarrow 0; w_w \rightarrow \infty; a_w \rightarrow a_s = \text{const.} \end{aligned}$$

w-sing. appear in: $f(R)$ gravity (Starobinsky '80), scalar field gravity (Setare, Saridakis '09), and brane gravity (Sahni, Shtanov '05)

w -duality



3. Classification and observational verification.

- Type 0 - Big-Bang (Big-Crunch) $a \rightarrow 0, p \rightarrow \infty, \rho \rightarrow \infty$
- Type I - Big-Rip $a(t_s) \rightarrow \infty (t_s < \infty), p \rightarrow \infty, \rho \rightarrow \infty$ (Caldwell 2002)
- Type II - Sudden Future (includes Big Boost and Big-Brake) $a(t_s) = \text{const.}, \rho = \text{const.}, p \rightarrow \infty$ (Barrow 2004)
- Type IIg - Generalized Sudden Future $a(t_s) = \text{const.}, \rho = \text{const.}, p = \text{const.}, \ddot{a} \rightarrow \infty$ etc., $w < \infty$ (Barrow 2004)
- Type III - Finite Scale Factor (also Big-Freeze) $a(t_s) = \text{const.}, \rho \rightarrow \infty, p \rightarrow \infty$ (NOT 2005, Denkiewicz 2012)
- Type IV - Big Separation: $a(t_s) = \text{const.}, p = \rho = 0, w \rightarrow \infty, \ddot{a} \rightarrow \infty$ etc. (NOT 2005) (and generalizations $p = \rho = \text{const.}$ Yurov 2010)
- Type V - w -singularity $a(t_s) = \text{const.}, p = \rho = 0, w \rightarrow \infty$ (MPD, Denkiewicz 2009) (and generalizations $p = \text{const.}$ Yurov 2010)
- Little-Rip $a(t_s) \rightarrow \infty, \rho(t_s) \rightarrow \infty (t_s \rightarrow \infty),$
- Pseudo-Rip $\rho(t_s) < \infty (t_s \rightarrow \infty)$ (Frampton et al. 2011, 2012)

Are these really singularities - strength?

As an example let us take an SFS which is determined by a **blow-up of the Riemann tensor** and its derivatives.

Geodesics **do not feel** SFS at all, since geodesic equations are not singular for $a_s = a(t_s) = \text{const.}$ (Fernandez-Jambrina, Lazkoz PRD 74, 064030 (2006))

$$\left(\frac{dt}{d\tau}\right)^2 = A + \frac{P^2 + KL^2}{a^2(t)}, \quad (21)$$

$$\frac{dr}{d\tau} = \frac{P_1 \cos\phi + P_2 \sin\phi}{a^2(t)} \sqrt{1 - Kr^2}, \quad (22)$$

$$\frac{d\phi}{d\tau} = \frac{L}{a^2(t)r^2}. \quad (23)$$

Geodesic deviation equation

$$\frac{D^2 n^\alpha}{d\lambda^2} + R^\alpha_{\beta\gamma\delta} u^\beta n^\gamma u^\delta = 0, \quad (24)$$

do feel SFS since at $t = t_s$ we have the Riemann tensor $R^\alpha_{\beta\gamma\delta} \rightarrow \infty$.

Classification of exotic singularities - strength.

- **Tipler's** (Phys. Lett. A64, 8 (1977)) definition (of a strong singularity):

$$\int_0^\tau d\tau' \int_0^{\tau'} d\tau'' |R_{ajb}^i u^a u^b|$$

diverges on the approach to a singularity at $\tau = \tau_s$

- i.e. an extended object is **crushed to zero volume** (represented by three linearly independent, vorticity-free geodesic deviation vectors at p along causal geodesic l) at the singularity by infinite tidal forces

- **Królak's** (CQG 3, 267 (1988)) definition (of a strong singularity):

$$\int_0^\tau d\tau' |R_{ajb}^i u^a u^b|$$

diverges on the approach to a singularity at $\tau = \tau_s$

- i.e. the **expansion** of every future-directed congruence of null (timelike) geodesics emanating from point p and containing l **becomes negative** somewhere on l

Classification of exotic singularities - strength.

Fernandez-Jambrina (PRD 82, 124004 (2010)) used **Puiseux series** expansion

$$a(t) = c_0 + (t_s - t)^{\eta_0} + c_1 (t_s - t)^{\eta_1} + c_2 (t_s - t)^{\eta_2} + \dots \quad \eta_0 < \eta_1 < \dots \quad c_0 > 0 \quad (25)$$

to check **the strength** of exotic singularities (T - Tipler; K - Królak)

Balcerzak and MPD (2006) considered **classical Polyakov strings**

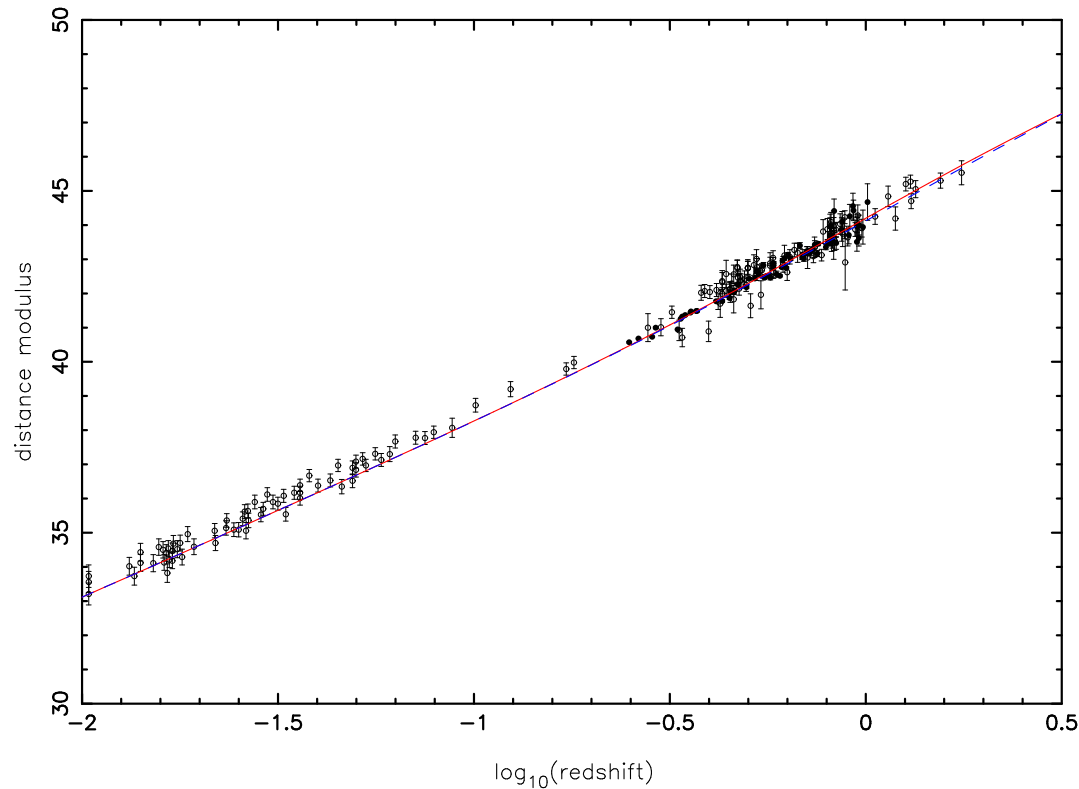
$$S = -\frac{T}{2} \int d\tau d\sigma \eta^{ab} g_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \quad (26)$$

with an invariant size $S(\tau) = 2\pi a(\eta(\tau))R(\tau)$ (circular ansatz with radius R) falling into exotic singularities to show that they are: **infinitely stretched** $S \rightarrow \infty$ **at Big-Rip** while **for SFS** the scale factor **is finite** at η -time at SFS so that **the invariant string size is also finite**. The same is true for type III, IV and GSFS. This means strings are **not destroyed** at these weak singularities.

Classification of singularities in FRW cosmology.

Type	Name	t sing.	$a(t_s)$	$\rho(t_s)$	$p(t_s)$	$\dot{p}(t_s)$ etc.	$w(t_s)$	T	K
0	Big-Bang (BB)	0	0	∞	∞	∞	finite	strong	strong
I	Big-Rip (BR)	t_s	∞	∞	∞	∞	finite	strong	strong
I_l	Little-Rip (LR)	∞	∞	∞	∞	∞	finite	strong	strong
I_p	Pseudo-Rip (PR)	∞	∞	finite	finite	finite	finite	weak	weak
II	Sudden Future (SFS)	t_s	a_s	ρ_s	∞	∞	finite	weak	weak
II_g	Gen. Sudden Future (GSFS)	t_s	a_s	ρ_s	p_s	∞	finite	weak	weak
III	Finite Scale Factor (FSF)	t_s	a_s	∞	∞	∞	finite	weak	strong
IV	Big-Separation (BS)	t_s	a_s	0	0	∞	∞	weak	weak
V	w-singularity (w)	t_s	a_s	0	0	0	∞	weak	weak

Exotic singularities can mimic dark energy.



SFS - supernovae only (MPD et al. 2007): distance modulus $\mu_L = m - M$ for the CC model ($H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_{m0} = 0.26$, $\Omega_{\Lambda 0} = 0.74$) (dashed curve) and SFS model ($m = 2/3 = 0.6666$, $n = 1.9999$, $\delta = -0.471$, $y_0 = 0.99936$ - **SFS in 8.7 mln years**) (solid curve). Open circles - ‘Gold’ data; filled circles -

SNI S data

CMB shift parameter.

It is possible to fit other tests but at the expense of **relaxing the range** of the parameter m which refers to Big-Bang limit ($m = 2/3$ is dust).

Shift parameter is:

$$\mathcal{R} = \frac{l_1'^{TT}}{l_1^{TT}} \quad (27)$$

where

l_1^{TT} – the temperature perturbation CMB spectrum multipole of the **first acoustic peak** in SFS model

$l_1'^{TT}$ – the multipole of a reference flat standard Cold Dark Matter model.

One usually uses a rescaled shift parameter:

$$\mathcal{R} = \frac{H_0 a_0}{c} \sqrt{\Omega_{m0}} r_{dec} = \sqrt{\Omega_{m0}} a'(y) \int_{y_{dec}}^{y_0} \frac{dy}{a(y)} = \sqrt{\Omega_{m0}} \int_0^{z_{dec}} \frac{dz}{E(z)}, \quad (28)$$

and WMAP data gives $\mathcal{R} = 1.70 \pm 0.03$ (Wang et al. 2006).

Baryon acoustic oscillations.

This can be done by measuring the transverse extend of an object (using the angular diameter distance $d_A = l/\Delta\theta$, where l is the linear size of an object) and the line-of-sight extend (using the redshift distance $\Delta x = c\Delta t/a(t) = ct_s\Delta y/a(y)$) (see e.g. Nesseris et al. 2006). As a result one defines the **volume distance** as

$$D_V^3 = d_A^2 \Delta x \quad , \quad (29)$$

so that one has

$$D_V = \left[\left(\int_{y_1}^{y_0} \frac{ct_s dy}{a(y)} \right)^2 \left(\frac{ct_s \Delta y}{a(y)} \right) \right]^{\frac{1}{3}} = \left[\left(\frac{c}{a_0 H_0} \int_0^z \frac{dz}{E(z)} \right)^2 \left(\frac{c}{a_0 H_0} \frac{\Delta z}{E(z)} \right) \right]^{\frac{1}{3}} . \quad (30)$$

Eisenstein et al. (2005) gave $D_V(\Delta z = z_{BAO} = 0.35) = 1370 \pm 64$ Mpc (an acoustic peak for 46748 luminous red galaxies (LRG) selected from the SDSS.

Baryon acoustic oscillations - dimensionless parameter \mathcal{A} .

For SFS models (8) it is more convenient to use a **dimensionless quantity** \mathcal{A} which is obtained multiplying D_V by $\sqrt{\Omega_{m0}}/(ct_s z_{BAO})$ or by $\sqrt{\Omega_{m0}}(a_0 H_0)/(cz_{BAO})$ to get

$$\mathcal{A} = \sqrt{\Omega_{m0}} a'(y_0) \left[\frac{a(y_{BAO})}{a'(y_{BAO}) a(y_0)} \right]^{\frac{1}{3}} \left[\frac{1}{z_{BAO}} \int_{y_{BAO}}^{y_0} \frac{dy}{a(y)} \right]^{\frac{2}{3}} \quad (31)$$

or

$$\mathcal{A} = \sqrt{\Omega_{m0}} E(z_{BAO})^{-1/3} \left[\frac{1}{z_{BAO}} \int_0^{z_1} \frac{dz}{E(z)} \right]^{2/3} \quad (32)$$

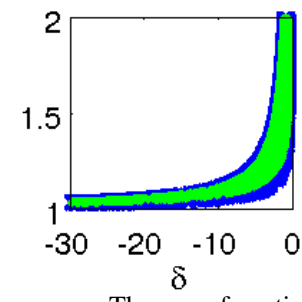
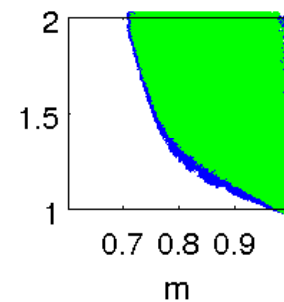
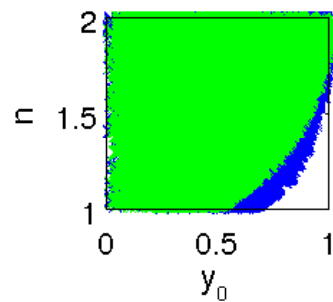
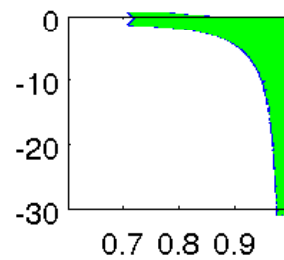
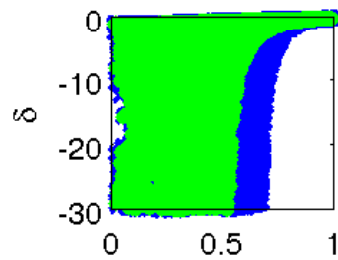
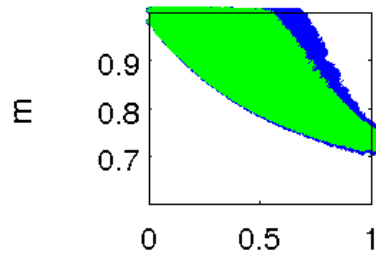
It should have the value (Eisenstein et al. 2005)

$$\mathcal{A} = 0.469 \left(\frac{n}{0.98} \right)^{-0.35} \pm 0.017 , \quad (33)$$

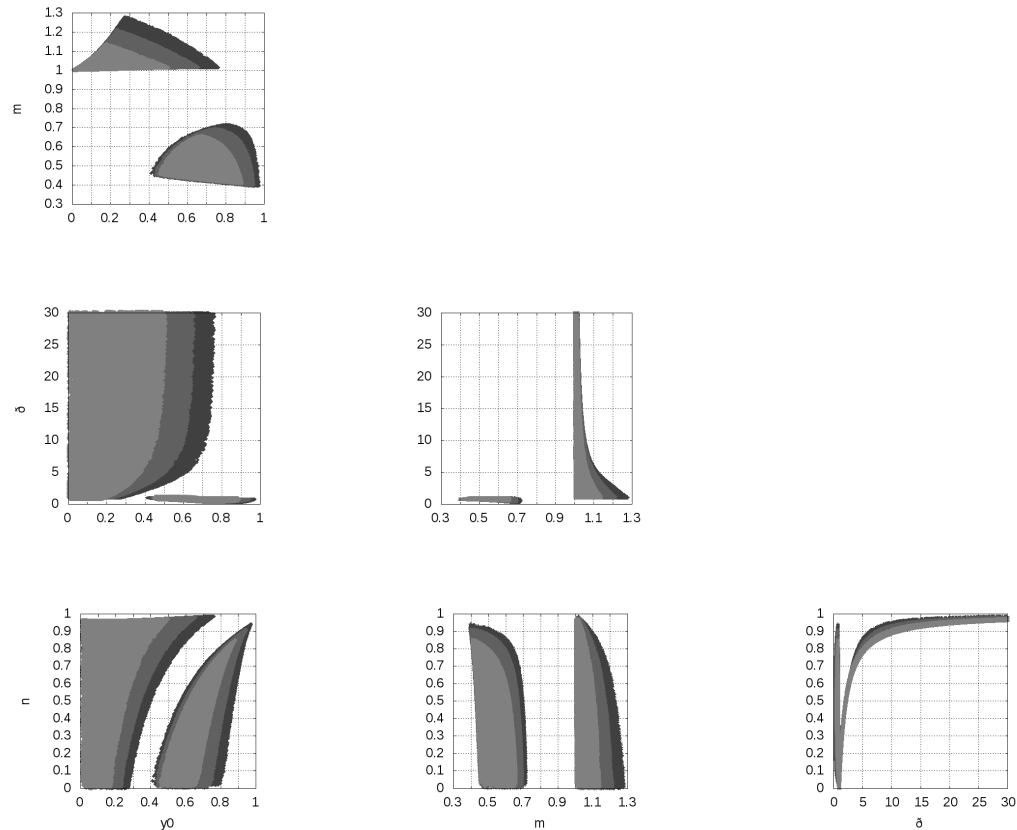
where n is the spectral index (now taken about ~ 0.96).

SFS: supernovae, CMB, BAO (Denkiewicz et al. 2012)

Fits if $m \approx 0.72$, $w = -0.82$



FSF: supernovae, CMB, BAO (Denkiewicz 2012)

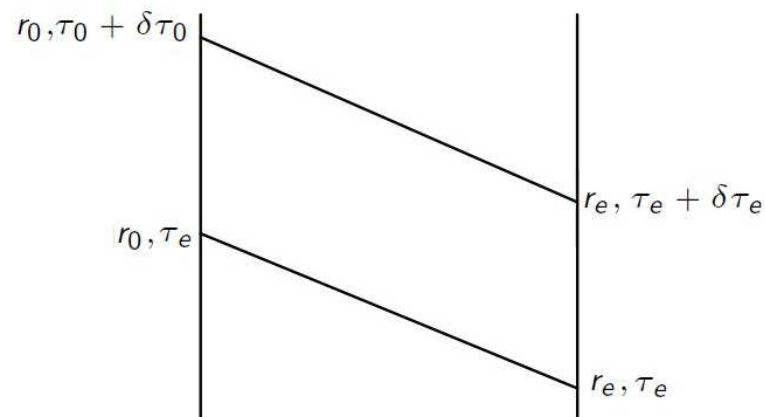


$m = 2/3$ (dust) matter allowed in the past; FSF **may happen** in $2 \cdot 10^9$ years in future (stronger, and closer to big-bang since $a = \text{const}$, and big-bang has $a = 0$).

Keresztes, et al. (2009, 2010) found similar value for the **Big-Brake model** which

Redshift drift test.

Redshift drift (Sandage 1962) - the idea is to collect data from two light cones separated by 10-20 years to look for a change in redshift of a source as a function of time.



There is a relation between the times of emission of light by the source τ_e and $\tau_e + \Delta\tau_e$ and times of their observation at τ_o and $\tau_o + \Delta\tau_o$:

$$\int_{\tau_e}^{\tau_o} \frac{d\tau}{a(\tau)} = \int_{\tau_e + \Delta\tau_e}^{\tau_o + \Delta\tau_o} \frac{d\tau}{a(\tau)}, \quad (34)$$

which for small $\Delta\tau_e$ and $\Delta\tau_o$ reads as $\frac{\Delta\tau_e}{a(\tau_e)} = \frac{\Delta\tau_o}{a(\tau_o)}$.

Redshift drift in exotic singularity models.

The redshift drift is defined as

$$\Delta z = z_e - z_0 = \frac{a(t_0 + \Delta t_0)}{a(t_e + \Delta t_e)} - \frac{a(t_0)}{a(t_e)}, \quad (35)$$

which can be expanded in series and to first order in Δt as

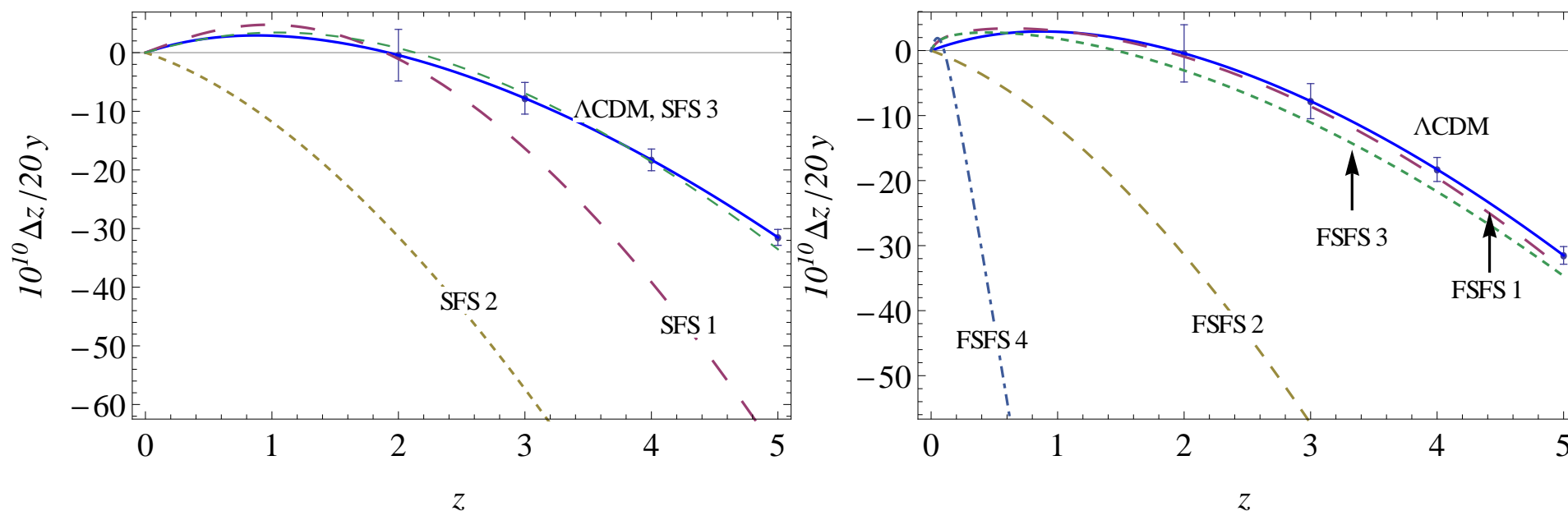
$$\begin{aligned} \Delta z &= \frac{a(t_0) + \dot{a}(t_0)\Delta t_0}{a(t_e) + \dot{a}(t_e)\Delta t_e} - \frac{a(t_0)}{a(t_e)} \\ &\approx \frac{a(t_0)}{a(t_e)} \left[\frac{\dot{a}(t_0)}{a(t_0)}\Delta t_0 - \frac{\dot{a}(t_e)}{a(t_e)}\Delta t_e \right]. \end{aligned} \quad (36)$$

Using above relations we have

$$\Delta z = \Delta t_0 [H_0(1+z) - H(t(z))] = (1+z) \frac{\Delta v}{c}, \quad (37)$$

where Δv is the velocity shift and $H(t(z))$ is given in a standard way.

Redshift drift for SFS, FSF (Denkiewicz, MPD, Martins, Vielzeuf, 2014).



SFS3, FSFS1, FSFS3 can **mimic** Λ CDM

SFS1, FSFS4 differ from Λ CDM significantly

SFS2, FSFS2 - dust Friedmann model

$H_0 = 67.3 \text{ km/s/Mpc}$ and $\Omega_{m0} = 0.315$ (Planck 2013).

redshift drift - parameters

	m	δ	n	t_0/t_s
SFS 1	2/3	-0.43	1.9999	0.99
SFS 2	2/3	0.0	1.9999	0.99
SFS 3	0.749	-0.45	1.99	0.77
FSFS 1	0.56	0.42	0.8	0.96
FSFS 2	2/3	0.0	0.7	0.79
FSFS 3	2/3	0.24	0.7	0.96
FSFS 4	1.15	7.5	0.81	0.51

RD [planned to be measured](#) by ELT-HIRES high-resolution ultra-stable spectrograph for the E-ELT (European Extermely Large Telescope) - Lyman- α forest. Also SKA (Square Kilometre Array), CHIME (The Canadian Hydrogen Intensity Mapping Experiment). Plus DECIGO/BBO - grav. wave related measurements.

4. Averaging approach - the universe through an exotic singularity.

A.K. Raychaudhuri (PRL 80, 654 (1998)) proposed that one may **average physical and kinematical scalars** over the whole open spacetime (provided they vanish rapidly at spatial and temporal infinity) as follows

$$\langle \chi \rangle = \lim_{x^a \rightarrow \infty} \frac{\int \int \int \int_{-x^a}^{x^a} \chi \sqrt{-g} d^4 x}{\int \int \int \int_{-x^a}^{x^a} \sqrt{-g} d^4 x} \quad (38)$$

By an open model it is meant that the ratio of the 3-volume hypersurfaces to a 4-volume of spacetime vanishes, i.e.,

$$\frac{\int \int \int \sqrt{|^3 g|} d^3 x}{\int \int \int \int \sqrt{-g} d^4 x} = 0. \quad (39)$$

His idea was to tigt the **vanishing** of the average $\langle \chi \rangle$ with the **singularity avoidance** in cosmology.

Spacetime averaging - density and pressure.

For the pressure, the energy density, and the average acceleration we have

$$\langle p \rangle = - \lim_{\substack{t_0 \rightarrow 0 \\ t_1 \rightarrow \infty}} \frac{\int_{t_0}^{t_1} a^3 \left(2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt} \quad (40)$$

and

$$\langle \rho \rangle = \lim_{\substack{t_0 \rightarrow 0 \\ t_1 \rightarrow \infty}} \frac{3 \int_{t_0}^{t_1} a^3 \left(\frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt}. \quad (41)$$

$$\langle \dot{\theta} \rangle = \lim_{\substack{t_0 \rightarrow 0 \\ t_1 \rightarrow \infty}} \frac{3 \int_{t_0}^{t_1} a^3 \left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt}. \quad (42)$$

SFS universe through an exotic singularity.

One is able to construct a **hybrid model** which allows Big-Bang, SFS, and finally Big-Crunch given by:

$$a_L(t) = a_s \left[\delta + \left(1 + \frac{t}{t_B}\right)^m (1 - \delta) - \delta \left(-\frac{t}{t_B}\right)^n \right] \quad (43)$$

with $t_B < 0$ - the Big-Bang time, and $t = 0$ and SFS time;

$$a_R(t) = a_s \left[\delta + \left(1 - \frac{t}{t_C}\right)^m (1 - \delta) - \delta \left(\frac{t}{t_C}\right)^n \right] \quad (44)$$

with $t_C > 0$ - the Big-Crunch time. In the high pressure regime $t \rightarrow 0$ these are approximated by

$$a_L \approx a_s \left[1 + \frac{m}{t_B} (1 - \delta) t \right], \quad (45)$$

$$a_R \approx a_s \left[1 - \frac{m}{t_C} (1 - \delta) t \right]. \quad (46)$$

Spacetime averaging - standard and phantom models

$$\langle p \rangle_{stand} = \lim_{\substack{t_0 \rightarrow 0 \\ t_1 \rightarrow \infty}} -\frac{4}{\gamma} \left(\frac{1}{\gamma} - 1 \right) \frac{\int_{t_0}^{t_1} t^{2(\frac{1}{\gamma}-1)} dt}{\int_{t_0}^{t_1} t^{\frac{2}{\gamma}} dt} \rightarrow 0,$$

$$\langle \rho \rangle_{stand} = \lim_{\substack{t_0 \rightarrow 0 \\ t_1 \rightarrow \infty}} -\frac{4}{3\gamma^2} \frac{\int_{t_0}^{t_1} t^{2(\frac{1}{\gamma}-1)} dt}{\int_{t_0}^{t_1} t^{\frac{2}{\gamma}} dt} \rightarrow 0$$

$$\langle p \rangle_{ph} = \lim_{\substack{t_0 \rightarrow 0 \\ t_1 \rightarrow \infty}} -\frac{4}{|\gamma|} \left(\frac{1}{|\gamma|} + 1 \right) \frac{\int_{t_0}^{t_1} t^{-2(\frac{1}{|\gamma|}+1)} dt}{\int_{t_0}^{t_1} t^{-\frac{2}{|\gamma|}} dt} \rightarrow \infty,$$

$$\langle \rho \rangle_{ph} = \lim_{\substack{t_0 \rightarrow 0 \\ t_1 \rightarrow \infty}} -\frac{4}{3\gamma^2} \frac{\int_{t_0}^{t_1} t^{-2(\frac{1}{|\gamma|}+1)} dt}{\int_{t_0}^{t_1} t^{-\frac{2}{|\gamma|}} dt} \rightarrow \infty$$

Spacetime averaging - SFS and FSF models

$$\dot{a}_L(t) = a_s \left[\frac{m}{t_B} \left(1 + \frac{t}{t_B} \right)^{m-1} (1 - \delta) + \delta \frac{n}{t_B^n} (-t)^{n-1} \right] \quad (47)$$

$$\dot{a}_R(t) = a_s \left[-\frac{m}{t_C} \left(1 - \frac{t}{t_C} \right)^{m-1} (1 - \delta) + \delta \frac{n}{t_C^n} (t)^{n-1} \right] \quad (48)$$

$$\frac{\ddot{a}_L}{a_s} = \frac{m(m-1)(1-\delta)}{t_B^2} \left(1 + \frac{t}{t_B} \right)^{m-2} - \frac{\delta n(n-1)}{t_B^n} (-t)^{n-2} \quad (49)$$

$$\frac{\ddot{a}_R}{a_s} = \frac{m(1-m)(1-\delta)}{t_C^2} \left(1 - \frac{t}{t_C} \right)^{m-2} + \frac{\delta n(n-1)}{t_C^n} t^{n-2} \quad (50)$$

Only the last terms blow up to give infinite pressure for $1 < n < 2$ at $t = 0$ so that we neglect other terms in a , \dot{a} and \ddot{a} .

Spacetime averaging - SFS and FSF models

$$\begin{aligned}
 \langle \dot{\theta} \rangle_{SFS,L} &= \lim_{\substack{t_0 \rightarrow -t_B \\ t_1 \rightarrow 0}} -3n \frac{\int_{t_0}^{t_1} (-t)^{3n-2} dt}{\int_{t_0}^{t_1} (-t)^{3n} dt} & (51) \\
 &= \lim_{\substack{t_0 \rightarrow -t_B \\ t_1 \rightarrow 0}} -3n \frac{3n+1}{3n-1} \frac{(-t_1)^{3n-1} - (-t_0)^{3n-1}}{(-t_1)^{3n+1} - (-t_0)^{3n+1}} \rightarrow \frac{1}{t_B^2}
 \end{aligned}$$

$$\begin{aligned}
 \langle \dot{\theta} \rangle_{SFS,R} &= \lim_{\substack{t_0 \rightarrow 0 \\ t_1 \rightarrow t_C}} 3n \frac{\int_{t_0}^{t_1} t^{3n-2} dt}{\int_{t_0}^{t_1} t^{3n} dt} & (52) \\
 &= \lim_{\substack{t_0 \rightarrow 0 \\ t_1 \rightarrow t_C}} 3n \frac{3n+1}{3n-1} \frac{t_1^{3n-1} - t_0^{3n-1}}{t_1^{3n+1} - t_0^{3n+1}} \rightarrow \frac{1}{t_C^2}
 \end{aligned}$$

These averages are finite for SFS, but they may blow up for FSF if $0 < n < 1/3$!

Subtle differences between singularities.

- BB, BC singularities - all the energy conditions fulfilled, averages vanish (despite original claim of Raychaudhuri)
- BR singularity - no EC fulfilled, averages blow up
- SFS - only dominant energy violated, averages finite
- It seems that BR is stronger singularity than BB, BC on the ground of averaging.
- SFS is weaker, but FSF does not seem so.

This seems to be a new kind of a measure for the strength of singularities.

4. Varying constants theories versus cosmic singularities.

- It has been shown that **quantum effects** (e.g. Houndjo 1008.0664; Houndjo et al. 1203.6084) may **change the strength** of exotic singularities (e.g. SFS can be changed into either FSF or BR or BC).
- On the other hand, varying constants cosmologies have been applied to **solve standard cosmology problems** such as the horizon and flatness problem (e.g. Albrecht, Magueijo 1999).
- **Our idea** is to apply them to **solve the singularity problem** in cosmology.
- We can also ask if varying constants theories **can soften/strengthen** the standard and exotic singularities?

varying constants theories

First fully quantitative framework: **Brans-Dicke** scalar-tensor gravity (1961)

The gravitational constant G is associated with an average gravitational potential (scalar field) ϕ surrounding a given particle:

$\langle \phi \rangle = GM/(c/H_0) \propto 1/G = 1.35 \times 10^{28} g/cm$. The **scalar field gives the strength of gravity**

$$G = \frac{1}{16\pi\Phi} \quad (53)$$

With the action

$$S = \int d^4x \sqrt{-g} \left(\Phi R - \frac{\omega}{\Phi} \partial_\mu \Phi \partial^\mu \Phi + \Lambda + L_m \right) \quad (54)$$

it relates to low-energy-effective **superstring** theory for $\omega = -1$

String coupling constant (running) $g_s = \exp(\phi/2)$ changes in time with ϕ - the **dilaton** and $\Phi = \exp(-\phi)$.

varying constants theories

Varying speed of light theories (VSL): Albrecht & Magueijo model (AM model) (1999)(Barrow 1999; Magueijo 2003):

$$c^4 = \psi(x^\mu) \quad (55)$$

and so the action is

$$S = \int d^4x \sqrt{-g} \left[\frac{\psi(R + 2\Lambda)}{16\pi G} + L_m + L_\psi \right] \quad (56)$$

AM model **breaks Lorentz invariance** (relativity principle and light principle) - preferred frame (cosmological or CMB) in which the field is minimally coupled to gravity.

Solves basic problems of standard cosmology: horizon problem and flatness problem.

Ansatz: Friedmann with $\rho = \rho_0 a^{-3\gamma}$, $c(t) = c_0 a^n$ - solution if $n \leq (1/2)(2 - 3\gamma)$.

varying constants theories

Magueijo covariant (conformally) and **locally invariant** model (2000, 2001):

$$\psi = \ln \left(\frac{c}{c_0} \right) \quad \text{or} \quad c = c_0 e^\psi, \quad (57)$$

with the action

$$S = \int d^4x \sqrt{-g} \left[\frac{c_0^4 e^{\alpha\psi} (R + 2\Lambda + L_\psi)}{16\pi G} + e^{\beta\psi} L_m \right], \quad (58)$$

with

$$L_\psi = \kappa(\psi) \nabla_\mu \psi \nabla^\mu \psi. \quad (59)$$

Further assumption: $\alpha - \beta = 4$.

Interesting subcases:

$\alpha = 4; \beta = 0$ - Brans-Dicke with $\phi_{BD} = e^{4\psi}/G$ and $\kappa(\psi) = 16\omega_{BD}(\phi_{BD})$.

$\alpha = 0; \beta = -4$ - minimal VSL theory.

varying constants theories

Varying fine structure constant α (or charge $e = e_0\epsilon(x^\mu)$) theories (Webb et al. 1999, Sandvik 2002)

$$S = \int d^4x \sqrt{-g} \left(\psi R - \frac{\omega}{2} \partial_\mu \psi \partial^\mu \psi - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} e^{-2\psi} + L_m \right) \quad (60)$$

with $\psi = \ln \epsilon$ and $f_{\mu\nu} = \epsilon F_{\mu\nu}$.

Assume linear expansion $e^\psi = 1 - 8\pi G \zeta (\psi - \psi_0) = 1 - \Delta\alpha/\alpha$ with the constraint on the local equivalence principle violation $|\zeta| \leq 10^{-3}$. [The relation to dark energy is:](#)

$$\gamma = w + 1 = \frac{(8\pi G \frac{d\psi}{d \ln a})^2}{\Omega_\psi}. \quad (61)$$

While mimicking the dark energy this can be tested by spectrograph CODEX (COsmic Dynamics EXplorer) attached to the planned E-ELT (European Extremely Large Telescope) which measures the redshift drift (or Sandage-Loeb effect) for $2 < z < 5$ (Vielzeuf and Martins 2012).

varying constants versus cosmic singularities.

We consider the Friedmann universes in **varying speed of light (VSL)** theories and **varying gravitational constant G** theories as follows (ρ - mass density; $\varepsilon = \rho c^2(t)$ - energy density in $Jm^{-3} = Nm^{-2} = kgm^{-1}s^{-2}$)

$$\rho(t) = \frac{3}{8\pi G(t)} \left(\frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2} \right), \quad (62)$$

$$p(t) = -\frac{c^2(t)}{8\pi G(t)} \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2} \right), \quad (63)$$

and the energy-momentum “conservation law” is

$$\dot{\rho}(t) + 3\frac{\dot{a}}{a} \left(\rho(t) + \frac{p(t)}{c^2(t)} \right) = -\rho(t) \frac{\dot{G}(t)}{G(t)} + 3\frac{kc(t)\dot{c}(t)}{4\pi Ga^2}. \quad (64)$$

General form of the scale factor.

We use a general form of the scale factor (MPD, K. Marosek, JCAP 02 (2013), 012), which **admits big-bang, big-rip, sudden future, finite scale factor and w -singularities** and reads as

$$a(t) = a_s \left(\frac{t}{t_s} \right)^m \exp \left(1 - \frac{t}{t_s} \right)^n, \quad (65)$$

with the constants t_s, a_s, m, n . For $k = 0$ we have

$$\rho(t) = \frac{3}{8\pi G(t)} \left[\frac{m}{t} - \frac{n}{t_s} \left(1 - \frac{t}{t_s} \right)^{n-1} \right]^2, \quad (66)$$

$$p(t) = -\frac{c^2(t)}{8\pi G(t)} \left[\frac{m(3m-2)}{t^2} - 6\frac{mn}{tt_s} \left(1 - \frac{t}{t_s} \right)^{n-1} \right. \\ \left. + 3\frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s} \right)^{2(n-1)} + 2\frac{n(n-1)}{t_s^2} \left(1 - \frac{t}{t_s} \right)^{n-2} \right]. \quad (67)$$

The scale factor - parametrization.

For $m < 0$ we have **a big-rip singularity** - $a \rightarrow \infty, \rho \rightarrow \infty, p \rightarrow \infty$ at $t = 0$;

For $1 < n < 2$ we have **a sudden future singularity** (SFS) which appears at $t = t_s$ ($a = a_s, \rho = \text{const.}, p \rightarrow \infty$);

For $0 < n < 1$ we have **a (stronger) finite scale factor singularity** (FSF) at $t = t_s$ ($a = a_s, \rho \rightarrow \infty, p \rightarrow \infty$).

In fact, for $1 < n < 2$ only the last term in the pressure of the type $(1 - t/t_s)^{n-2}$ blows-up, while for $0 < n < 1$ two more terms $(1 - t/t_s)^{n-1}$ and $(1 - t/t_s)^{2(n-1)}$ do.

Regularizing singularities by varying constants

One bears in mind the scale factor (65), the energy density (66) and pressure (67)

Regularizing a Big-Bang singularity by varying G :

If

$$G(t) \propto \frac{1}{t^2} \quad (68)$$

which is a faster decrease than in Dirac's LNH $G \propto 1/t$, but influences less the temperature of the Earth constraint (Teller 1948).

Both divergence in ρ and p are removed, though **at the expense of having the "singularity" of strong gravitational coupling $G \rightarrow \infty$ at $t \rightarrow 0$.**

In the Dirac's case, only the ρ singularity can be removed.

regularizing singularities by varying constants: SFS

Regularizing an SFS singularity by varying c :

If

$$c(t) = c_0 \left(1 - \frac{t}{t_s}\right)^{\frac{p}{2}}, \quad (69)$$

then

$$p(t) = -\frac{c_0^2}{8\pi G} \left[\frac{m(3m-2)}{t^2} \left(1 - \frac{t}{t_s}\right)^p - 6\frac{mn}{tt_s} \left(1 - \frac{t}{t_s}\right)^{p+n-1} + 3\frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{p+2n-2} + 2\frac{n(n-1)}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{p+n-2} \right].$$

and the singularity of pressure is **regularized provided $p > 2 - n$, ($1 < n < 2$)**.

regularizing singularities by varying constants: SFS.

Physical consequence: **light eventually stops** at the singularity. Same happens in loop quantum cosmology (LQC) where it is called the **anti-newtonian limit** $c = c_0 \sqrt{1 - \rho/\rho_c} \rightarrow 0$ for $\rho \rightarrow \rho_c$ with ρ_c being the critical density (Calettau et al. 2012). The **low-energy limit** $\rho \ll \rho_0$ gives the standard limit $c \rightarrow c_0$.

It also appears naturally in **Magueijo model** ((Magueijo, PRD 63, 043502 (2001))) in which black holes are not reachable since the **light stops at the horizon** (despite they possess Schwarzschild singularity). An observer cannot reach this surface even in his finite proper time.

Strangely, both options $c = 0$ and $c = \infty$ are possible in Magueijo model.

regularizing singularities by varying constants: w -sing.

In the limit $m \rightarrow 0$ we have an exotic singularity scale factor given by $a(t) = a_s \exp(1 - t/t_s)$ and so from (66) and (67) we have

$$\rho_{ex}(t) = \frac{3}{8\pi G(t)} \frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{2(n-1)}, \quad (70)$$

$$p_{ex}(t) = -\frac{c^2(t)}{8\pi G(t)} \left[3 \frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{2(n-1)} + 2 \frac{n(n-1)}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{n-2} \right] \quad (71)$$

so that

$$w_{ex}(t) = \frac{p_{ex}(t)}{\varepsilon_{ex}(t)} = - \left[1 + \frac{2}{3} \frac{n-1}{n} \frac{1}{\left(1 - \frac{t}{t_s}\right)^n} \right] = - \left[\frac{1}{3} - \frac{2}{3} q_{ex}(t) \right], \quad (72)$$

which is a w -singularity for $n > 2$ ($p = \rho = 0$, $w_{ex} \rightarrow \infty$). Its regularization by varying $c(t)$ is impossible since there is no c -dependence here.

regularizing singularities by varying constants: SFS

Regularizing an SFS singularity by varying G :

If we assume that

$$G(t) = G_0 \left(1 - \frac{t}{t_s}\right)^{-r}, \quad (73)$$

($r = \text{const.}$, $G_0 = \text{const.}$) which changes (66) and (67) to

$$\begin{aligned} \varrho(t) &= \frac{3}{8\pi G_0} \left[\frac{m^2}{t^2} \left(1 - \frac{t}{t_s}\right)^r - \frac{2mn}{tt_s} \left(1 - \frac{t}{t_s}\right)^{r+n-1} \right. \\ &\quad \left. + \frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{r+2n-2} \right], \end{aligned} \quad (74)$$

$$\begin{aligned} p(t) &= -\frac{c^2}{8\pi G_0} \left[\frac{m(3m-2)}{t^2} \left(1 - \frac{t}{t_s}\right)^r - 6\frac{mn}{tt_s} \left(1 - \frac{t}{t_s}\right)^{r+n-1} + 3\frac{n^2}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{r+2n-2} \right. \\ &\quad \left. + 2\frac{n(n-1)}{t_s^2} \left(1 - \frac{t}{t_s}\right)^{r+n-2} \right]. \end{aligned} \quad (75)$$

regularizing singularities by varying constants: SFS

From (74) and (75) it follows that an SFS singularity ($1 < n < 2$) is **regularized** by varying gravitational constant when

$$r > 2 - n , \quad (76)$$

and an FSF singularity ($0 < 1 < n$) is **regularized** when

$$r > 1 - n . \quad (77)$$

On the other hand, assuming that we have an SFS singularity and that

$$-1 < r < 0 , \quad (78)$$

we get that varying G **may change an SFS singularity onto a stronger FSF singularity** when

$$0 < r + n < 1 . \quad (79)$$

Regularizing singularities: (anti-)Chaplygin gas

The equation of state of the (anti-)Chaplygin gas reads as

$$p(t) = \pm \frac{A}{\varepsilon(t)} = \pm \frac{A}{\rho(t)c^2(t)} \quad (A > 0) , \quad (80)$$

where the “-” sign is for Chaplygin gas while the “+” sign is for anti-Chaplygin gas case and the unit of A is the energy density(=pressure) square $J^2 m^{-6}$.

Inserting (80) into (64) gives

$$\dot{\rho}(t) + 3 \frac{\dot{a}}{a} \left(\frac{\rho^2 c^4(t) \mp A}{\rho(t) c^4(t)} \right) = -\rho(t) \frac{\dot{G}(t)}{G(t)} + 3 \frac{kc(t)\dot{c}(t)}{4\pi G(t)a^2} . \quad (81)$$

We assume both varying $G = G(t)$ and $c = c(t)$ though with zero curvature ($k = 0$) as follows

$$\rho(t)c^2(t) = B = \text{const.} , \quad (82)$$

regularizing singularities: (anti-)Chaplygin gas

The solution of (81) reads as

$$\varrho(t)a^{3\gamma}(t)G(t) = E = \text{const.} , \quad (83)$$

where we have defined

$$\gamma \equiv \frac{B^2 \mp A}{B^2} \quad (84)$$

Putting the standard big-bang scale factor $a(t) = (t/t_s)^{2/3\gamma}$, we finally have

$$\varrho(t) = \frac{Et_s^2}{t^2 G(t)} , \quad p(t) = \mp \frac{A}{B} = \text{const.} , \quad (85)$$

which give $\varrho \rightarrow \infty$ and $p(0) = 0$ provided $G(0) = \text{const.} \neq 0$. The singularity at $t = 0$ in ϱ and p **can be regularized** by taking $G(t) \propto 1/t^2$ at the expense of having a constant pressure (cosmological term) instead of zero pressure.

6. Conclusions

- Currently one is able to **differentiate quite a number of cosmological singularities** with completely different properties - despite many of them are geodesically complete, they still lead to a **blow-up of physical quantities** such as scale factor, energy density, pressure, physical fields etc.
- Some of these singularities **may serve as dark energy**. **SFS may even appear in near future (8.7 Myr)** while FSF in more distant future (2 Gyr). They **can be fitted** to a combined SnIa, CMB, BAO data and mimic Λ CDM in redshift drift effect for specific choice of the parameters. They appear in physical theories (string, brane, scalar fields, $f(R)$, LQC etc.).
- **The exotic singularities can be influenced by varying constants**. It is possible to **"regularize" (remove infinities) or change** the type of these singularities with full physical consequences of this. However, we may face **new "singularity" in a physical field** responsible for the variability of constants but this is what happens in physical theories (e.g. superstring) too.