Dark energy from timely and spatial singularities of pressure

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Content:

- 1. Phantom dark energy and the way to exotic singularities.
- 2. The zoo of exotic singularities in homogeneous cosmology.
- 3. Dark energy from timely singularities of pressure.
- 4. Singularities in inhomogeneous cosmology.
- 5. Dark energy from spatial singularities of pressure.
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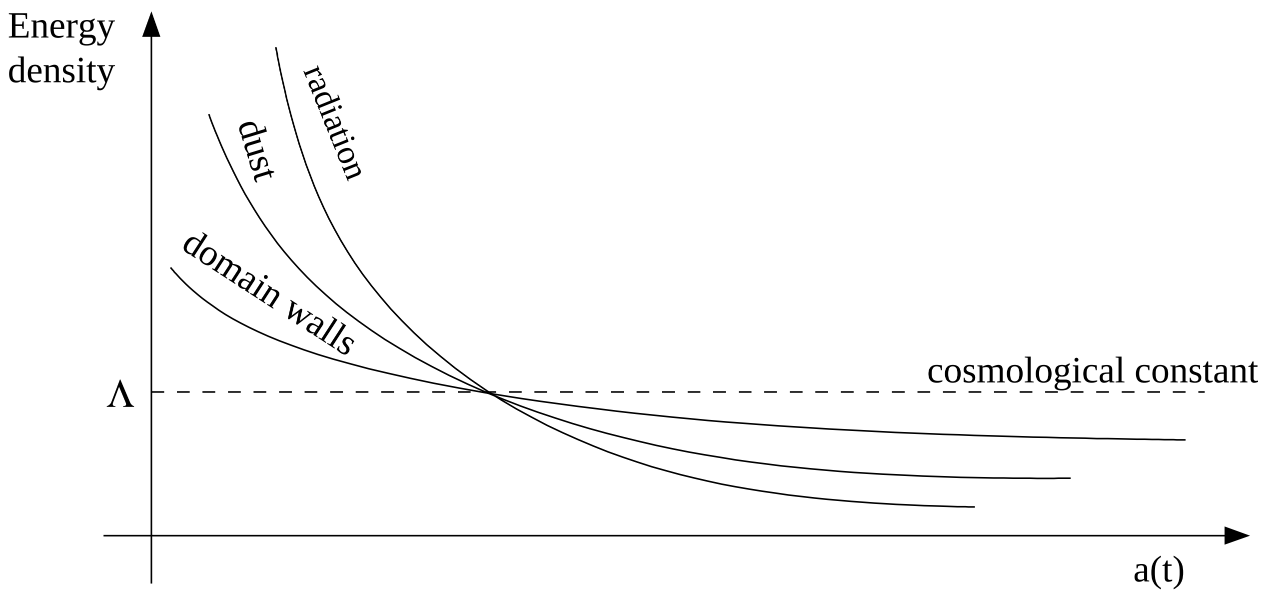
1. Phantom dark energy and the way to exotic singularities.

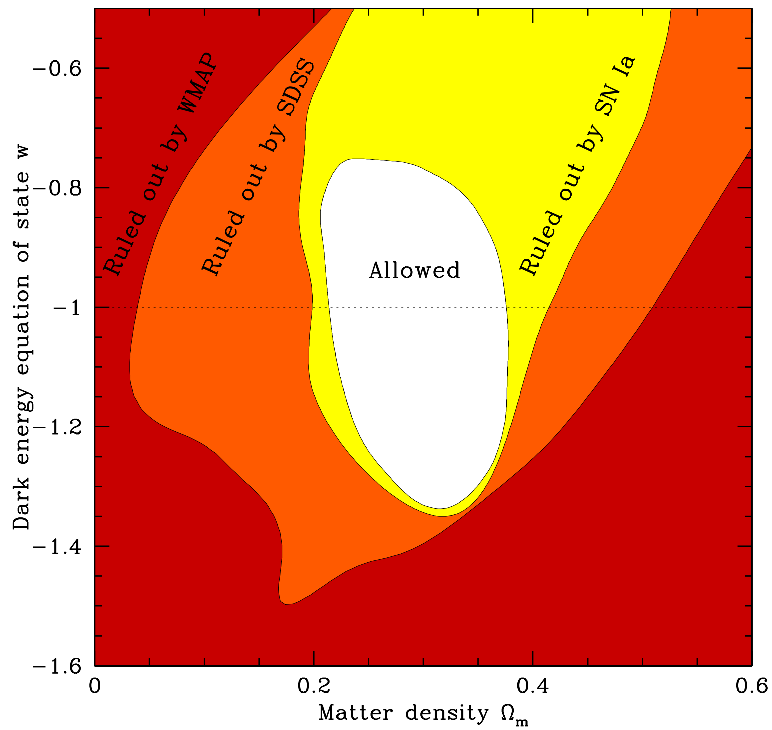
According to the cosmological no-hair theorem the dark energy with $w =$ const. ≥ -1 (violation of the strong energy condition only) will always be dominated by the cosmological constant ($w = -1$)



Then: any combination of dark energy with $w =$ const. ≥ -1 leads to "standard" Big-Bang/Crunch cosmological singularities: (or to emptiness - de Sitter).

Recent astronomical data shows possible violation of $w \geq -1$.

WMAP + SDSS + Supernovae **combined bound on the dark energy barotropic index** w (Tegmark et al. (2004)):



- **There is no sharp cut-off of the data at $p = -\varrho$!!!**
- **Dark energy with $p < -\varrho$ (phantom) can be admitted!**

More data:

- Knop et al. 2003 (from SNe + CMB + 2dFGRS combined) – $w = -1.05^{+0.15}_{-0.20}$ (statistical) ± 0.09 (systematic)
- Kowalski et al. (arXiv:0804.4142) analyzed 307 supernovae (Sne + BAO + CMB) – $w = -1.001^{+0.059}_{-0.063}$ (statistical) $^{+0.063}_{-0.066}$ (systematic)
- Can be considered as evidence for cosmic "no-hair" theorem violation - **even a small fraction of phantom dark energy will dominate the evolution**
- **Phantom** is dark energy of **a very large negative pressure** (Caldwell astro-ph/9908168 - published in PLB 2002) which violates null $(\varrho + p \geq 0)$, weak $(\varrho + p \geq 0,\ \rho \geq 0)$ and dominant energy $(\mid p \mid \leq \varrho,\ \varrho \geq 0)$ conditions.
- **Phantom dark energy is related to a new type of singularity in the universe – Big-Rip**

Big-Rip (type I) as an exotic singularity.

For convenience take

$$| w + 1 |= -(w + 1) > 0 \ , \qquad (1)$$

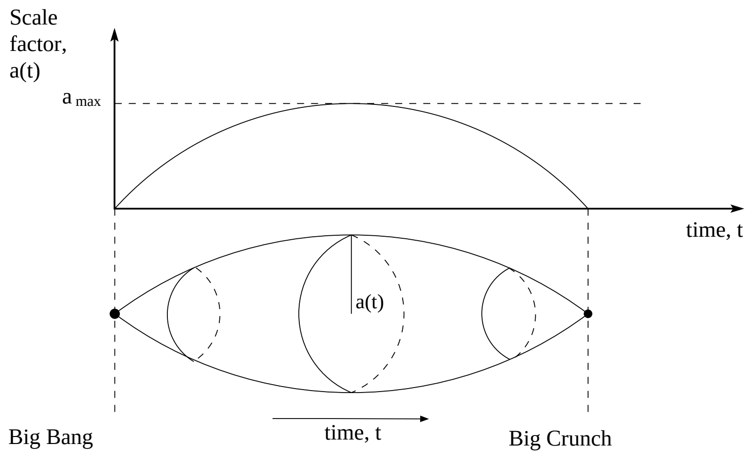
so that the conservation law for phantom gives

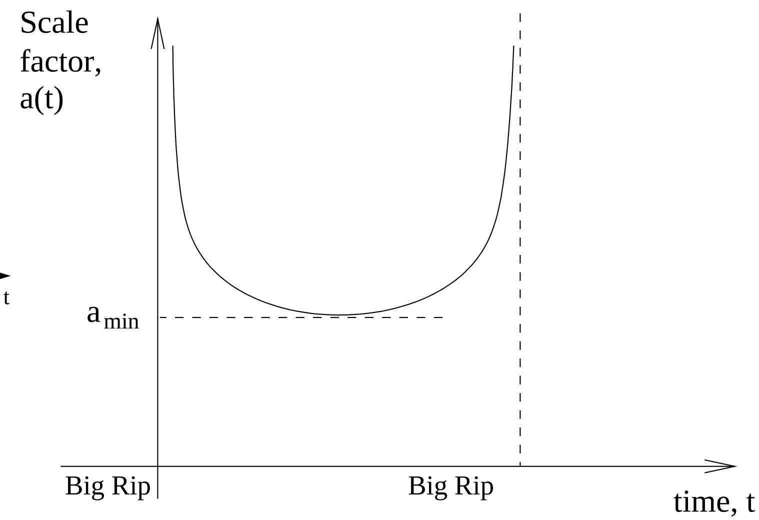
$$\varrho \propto a^{3|w+1|} \ . \qquad (2)$$

- Conclusion: the bigger the universe grows, the denser it is, and **it becomes dominated by phantom (overcomes Λ-term)** – an exotic future singularity appears – Big-Rip $\varrho, p \to \infty$ for $a \to \infty$
- Curvature invariants R^2, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ diverge at Big-Rip
- Only for $-5/3 < w < -1$ the null geodesics are geodesically complete; for other values of w, including all timelike geodesics, there is a geodesic incompleteness (Lazkoz et al. gr-qc/0607073, PRD '07) - the singularity is reached in a finite proper time.

Phantom duality

BR to BR model as dual to standard BB to BC model (MPD et al. 2003)





Duality: Standard matter ($p > -\varrho$) $\leftrightarrow$ Phantom ($p < -\varrho$)

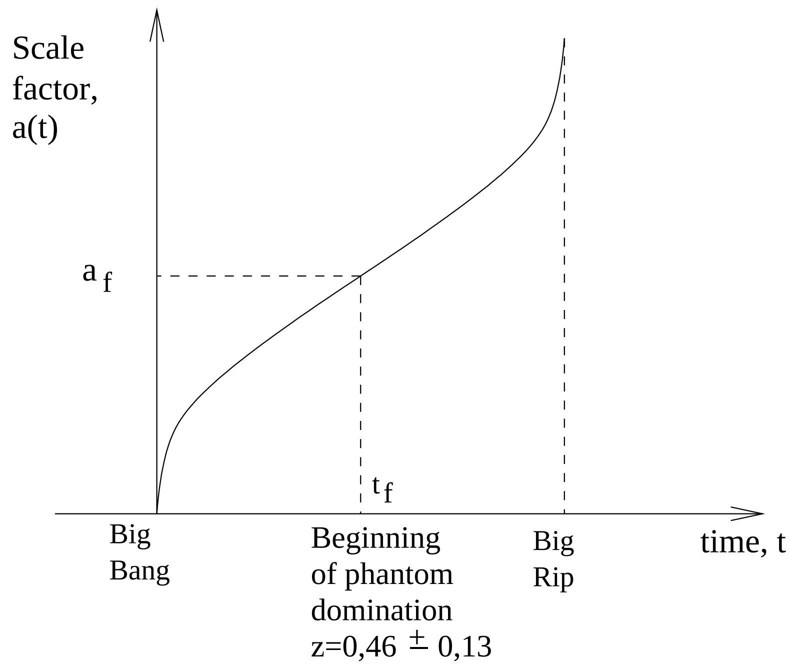
$$w \leftrightarrow -(w+2) \qquad \text{or better} \;\; \gamma \leftrightarrow -\gamma\,, \quad \gamma = w+1 \tag{3}$$

i.e.

$$a(t) \leftrightarrow \frac{1}{a(t)} \tag{4}$$

Phantom and Big-Rip-singularity-driven dark energy.

The universe begins with a Big-Bang and terminates at a Big-Rip:



so that standard Hot-Big-Bang cosmology is preserved (with a turning point at z = 0.46 (since $j_0 > 0$) Riess et al. 2004). Data shows that a Big -Rip is possible in the far future: $t = t_{BR} \approx 20$ Gyr.

Strange properties of a Big-Rip gave a push to studies some other exotic (or just "non-standard") types of singularities and the sources of dark energy which can be related to them.

2. The zoo of exotic singularities in homogeneous cosmology.

The studies of the basic Friedmann cosmology showed that there exist many different types of exotic singularities which are directly related to some sources of dark energy in the universe. These are:

- Sudden Future Singularity (type II) – SFS
- Finite Scale Factor (type III) Singularity – FSF
- Generalized Sudden Future Singularity – GSFS
- Big Separation (type IV) – BS
- $w-$singularity.

Sudden Future Singularity (type II) as an exotic singularity.

It is “exotic” in the sense that:

– it manifests as a singularity of pressure (or $\ddot{a}$) only

– it leads to the dominant energy condition violation only

The hint (Barrow ’04):

release the assumption about the imposition of an equation of state

$$p \neq p(\varrho), \qquad \text{no analytic form of this relation is given} \tag{5}$$

Choose a special form of the scale factor (may be motivated in fundamental cosmologies) as:

$$a(t) = a_s \left[\delta + (1-\delta)\, y^m - \delta\, (1-y)^n\right] \,, \qquad y \equiv \frac{t}{t_s} \tag{6}$$

where $a_s \equiv a(t_s) =$ const. and $\delta, m, n =$ const.

Obviously, for $t = 0$ one has a Big-Bang singularity.

There is a Big-Bang at $t = 0$ and a pressure singularity on a hypersurface $t = t_s$?

$$\dot{a} = a_s \left[\frac{m}{t_s} (1-\delta) y^{m-1} + \delta \frac{n}{t_s} (1-y)^{n-1} \right] , \tag{7}$$

$$\ddot{a} = \frac{a_s}{t_s^2} \left[m(m-1)(1-\delta) y^{m-2} - \delta n(n-1)(1-y)^{n-2} \right] . \tag{8}$$

If we assume that

$$1 < n < 2, \tag{9}$$

then using Einstein equations (38)-(40) we get

$$a = \text{const.}, \qquad \dot{a} = \text{const.} \qquad \varrho = \text{const.}$$
$$\ddot{a} \rightarrow \mp\infty \qquad p \rightarrow \pm\infty \qquad \text{for} \qquad t \rightarrow t_s \tag{10}$$

Friedmann limit - “nonstandardicity” parameter δ

Friedmann limit is easily obtained in the limit of “nonstandardicity” parameter $\delta \to 0$.

The parameter m can be taken to be just a form of the w parameter present in the barotropic equation of state:

- $0 < m \leq 1$ when $w \geq -1/3$ (standard matter);
- $m > 1$ when $-1 < w < -1/3$ (quintessence);
- $m < 0$ when $w < -1$ (phantom).

Near to SFS one has

$$a_{SFS} = a_s \left[1 - \delta (1-y)^n\right] \ , \tag{11}$$

and n plays the role in making a pressure blow-up.

Important point:

Unless we take $m > 1$ or $m < 0$ as an independent source of energy, acceleration is possible only for $\delta < 0$!

We will call it pressure-driven dark energy (whatever it is!).

Timely pressure singularity is a “weak” singularity

SFS are determined by a blow-up of the Riemann tensor and its derivatives

Geodesics do not feel SFS at all, since geodesic equations are not singular for $a_s = a(t_s) =$ const. (Fernandez-Jambrina, Lazkoz PRD 74, 064030 (2006) ((gr-qc/0607073))

$$\left(\frac{dt}{d\tau}\right)^2 = A + \frac{P^2 + KL^2}{a^2(t)} , \qquad (12)$$

$$\frac{dr}{d\tau} = \frac{P_1 cos\phi + P_2 \sin\phi}{a^2(t)}\sqrt{1 - Kr^2} , \qquad (13)$$

$$\frac{d\phi}{d\tau} = \frac{L}{a^2(t) r^2} . \qquad (14)$$

Geodesic deviation equation

$$\frac{D^2 n^\alpha}{d\lambda^2} + R^\alpha_{\beta\gamma\delta} u^\beta n^\gamma u^\delta = 0 , \qquad (15)$$

feels SFS since at $t = t_s$ we have the Riemann tensor $R^\alpha_{\beta\gamma\delta} \to \infty$.

No geodesic incompleteness.

- No geodesic incompleteness ($a =$ const. and r.h.s. of geodesic eqs. do not diverge) $\Rightarrow$ SFS are not the final state of the universe
- Point particles do not even see SFS while extended objects may suffer instantaneous infinite tidal forces but still may not be crushed.
- They are weak curvature singularities i.e. in-falling observers or detectors are not destroyed by tidal forces:
- Tipler's (Phys. Lett. A64, 8 (1977)) definition (of a weak singularity):
  $\int_0^\tau d\tau' \int_0^{\tau'} d\tau'' R_{ab} u^a u^b$
  does not diverge on the approach to a singularity at $\tau = \tau_s$
- Królak's (CQG 3, 267 (1988)) definition (of a weak singularity):
  $\int_0^\tau d\tau' R_{ab} u^a u^b$
  does not diverge on the approach to a singularity at $\tau = \tau_s$
- A singularity may be weak acc. to Tipler, but strong according to Królak
- Conclusion: a Big-Rip and an SFS are of different nature.

Type III (Finite Scale Factor - FSF) exotic singularity.

Type III singularities (Nojiri et al. PRD 71, 063004 (2005)) which we will call Finite Scale Factor singularities are characterized by the following conditions:

$$a = a_s = \text{const.},\ \varrho, \dot{a}_s \to \infty,\ |p|, \ddot{a}_s \to \infty$$

The simplest way to get them is to apply the scale factor as given previously for SFS, i.e.,

$$a(t) = a_s \left[\delta + (1-\delta)\, y^m - \delta\, (1-y)^n\right] , \qquad y \equiv \frac{t}{t_s} \tag{16}$$

where $a_f \equiv a(t_f) =$ const. and $\delta, A, m, n =$ const., but with the range of parameter n changed from $1 < n < 2$ into

$$0 < n < 1$$

FSF singularities are weak according to Tipler's definition, but strong according to Królak's.

Generalized Sudden Future singularity.

Sudden future singularities may be generalized to GSFS.

Take a general scale factor time derivative of an order r:

$$a^{(r)} = a_s \left[\frac{m(m-1)...(m-r+1)}{t_s^r} (1-\delta) \, y^{m-r} + (-1)^{r-1} \delta \frac{n(n-1)...(n-r+1)}{t_s^r} (1-y)^{n-r} \right] , \qquad (17)$$

Choosing (Barrow '04, Lake '04) $r-1 < n < r$, for any integer r we have a singularity in the scale factor derivative $a^{(r)}$, and consequently in the appropriate pressure derivative $p^{(r-2)}$.

Note that it fullfils all the energy conditions for any $r \geq 3$!!!

Big Separation (type IV)

Type IV singularity according to Nojiri et al. ’05 is when:

$$a = a_s = \text{const.},\ \varrho \to 0,\ p \to 0,\ \dddot{a},\ \ddot{H} \to \infty \text{ etc.}$$

and so it is similar to Generalized Sudden Future singularity with only one exception: it also gives the divergence of the barotropic index in the barotropic equation of state

$$p(t) = w(t)\varrho(t)$$

i.e.,

$$w(t) \to \infty$$

Barotropic index $w-$singularity

Another exotic is a $w-$singularity only (without the divergence of the higher-derivatives of the scale factor). (Apparently, it appears in physical theories such as $f(R)$ gravity (Starobinsky '80), scalar field gravity (Setare, Saridakis '09, and brane gravity Sahni, Shtanov '05)). We choose

$$a(t) = A + B\left(\frac{t}{t_s}\right)^{\frac{2}{3\gamma}} + C\left(D - \frac{t}{t_s}\right)^{n} , \tag{18}$$

where A, B, C, D, γ, n, and t_s are constants and impose the conditions:

$$a(0) = 0,\ a(t_s) = const. \equiv a_s,\ \dot{a}(t_s) = 0,\ \ddot{a}(t_s) = 0 , \tag{19}$$

which finally leads to the following form of the scale factor:

$$
\begin{aligned}
a(t) \quad &= \quad \frac{a_s}{1-\frac{3\gamma}{2}\left(\frac{n-1}{n-\frac{2}{3\gamma}}\right)^{n-1}} \\
&+ \quad \frac{1-\frac{2}{3\gamma}}{n-\frac{2}{3\gamma}}\frac{na_s}{1-\frac{2}{3\gamma}\left(\frac{n-\frac{2}{3\gamma}}{n-1}\right)^{n-1}}\left(\frac{t}{t_s}\right)^{\frac{2}{3\gamma}} \\
&+ \quad \frac{a_s}{\frac{3\gamma}{2}\left(\frac{n-1}{n-\frac{2}{3\gamma}}\right)^{n-1}-1}\left(1-\frac{1-\frac{2}{3\gamma}}{n-\frac{2}{3\gamma}}\frac{t}{t_s}\right)^{n} , \qquad (20)
\end{aligned}
$$

with the admissible values of the parameters: $\gamma > 0$ and $n \neq 1$.

$w-$duality

We have a blow-up of the effective barotropic index, i.e.,

$$w(t_s) = \frac{c^2}{3}\left[2q(t_s) - 1\right] \to \infty \; , \tag{21}$$

accompanied by

$$p(t_s) \to 0; \quad \varrho(t_s) \to 0 \; . \tag{22}$$

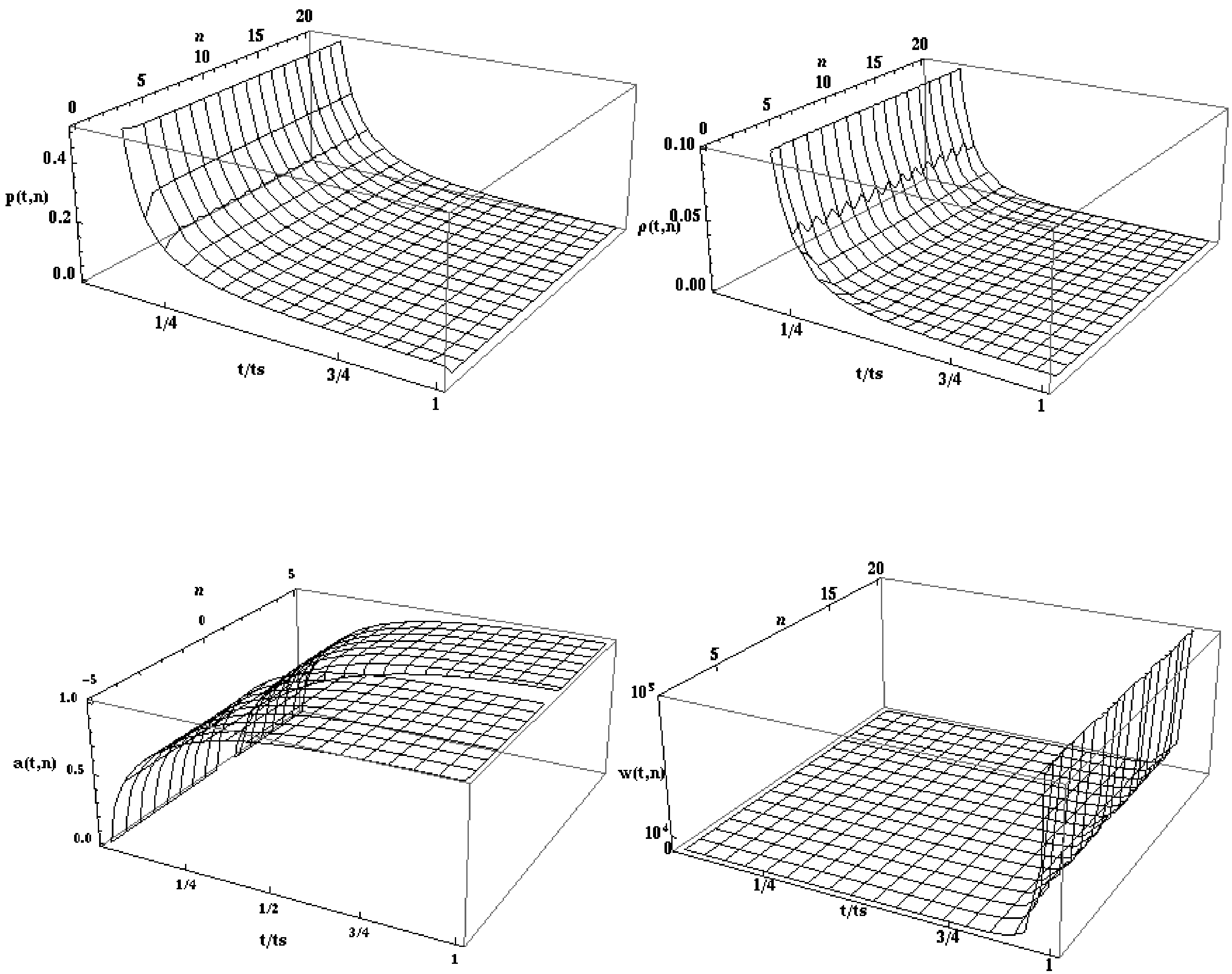
There is an amazing duality between the Big-Bang and the w-singularity in the form

$$p_{BB} \leftrightarrow \frac{1}{p_w}, \quad \varrho_{BB} \leftrightarrow \frac{1}{\varrho_w}, \quad w_{BB} \leftrightarrow \frac{1}{w_w} \quad . \tag{23}$$

In other words:

$$p_{BB} \to \infty; \, \varrho_{BB} \to \infty; \, w_{BB} \to 0; \, a_{BB} \to 0$$
$$p_w \to 0; \, \varrho_w \to 0; \, w_w \to \infty; \, a_w \to a_s = \text{const.}$$

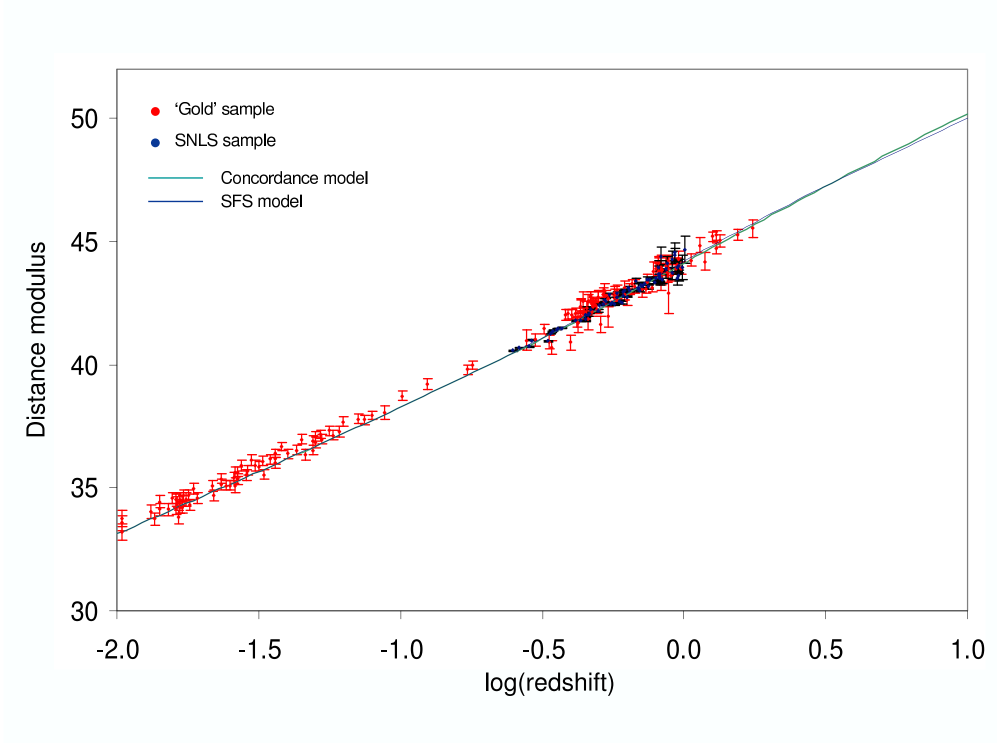
$w-$duality



3. Dark energy from timely singularities of pressure.

- Though not exemplified explicitly so far, it is the fact that there is usually some fundamental physical theory (scalar field, higher-order, string, brane, LQC) which can be related to the models with exotic singularities.
- In other words, the evidence for an exotic singularity may be attached to some form of matter which gives current acceleration of the universe and makes a candidate for the dark energy.
- The best studied models are of course phantom models which still are within the range of observational limit.
- However, we will show that some other models (in particular SFS models) can play a good candidate for the dark energy, too.

Timely pressure dark energy versus Λ-term dark energy (concordance cosmology - CC)



Distance modulus $\mu_L = m - M$ for the CC model ($H_0 = 72\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$, $\Omega_{m0} = 0.26$, $\Omega_{\Lambda 0} = 0.74$) (dashed curve) and SFS model ($m = 2/3, n = 1.9999, \delta = -0.471, y_0 = 0.99936$) (solid curve). Open circles are for the 'Gold' data and filled circles are for SNLS data.

Timely pressure singularities dark energy surprise.

Surprising remark:
If the age of the SFS model is equal to the age of the CC model, i.e. $t_0 = 13.6$ Gyr, one finds that **an SFS is possible in only 8.7 million years!!!**.

- In this context it is no wonder that the singularities were termed “sudden”.
- It was checked that GSFS (generalized SFS - no energy conditions violation) are always more distant in future. That means the strongest of SFS type singularities is more likely to become reality.
- A practical tool to recognize them well in advance is to measure possible large values of statefinders (deceleration parameter, jerk, snap etc.)!

Interesting point: SFS plague loop quantum cosmology! - see Wands et al. PRL ’08 ;arXiv: 0808.0190.

Big-Brake-exotic-singularity-driven dark energy.

SFS with $a = a_b$ = const., $\dot{a} = 0$ ($\varrho \to 0$), and $\ddot{a} \to -\infty$ ($p \to \infty$) were also termed Big-Brake (Gorini, Kamenschchik et al. PRD 69 (2004), 123512). They fulfill an anti-Chaplygin gas equation of state of the form

$$p = \frac{A}{\varrho} \qquad \mathrm{A} = \mathrm{const.} \quad . \tag{24}$$

They were studied in the context of the tachyon cosmology by Gergely, Keresztes et al. (0901.2292).

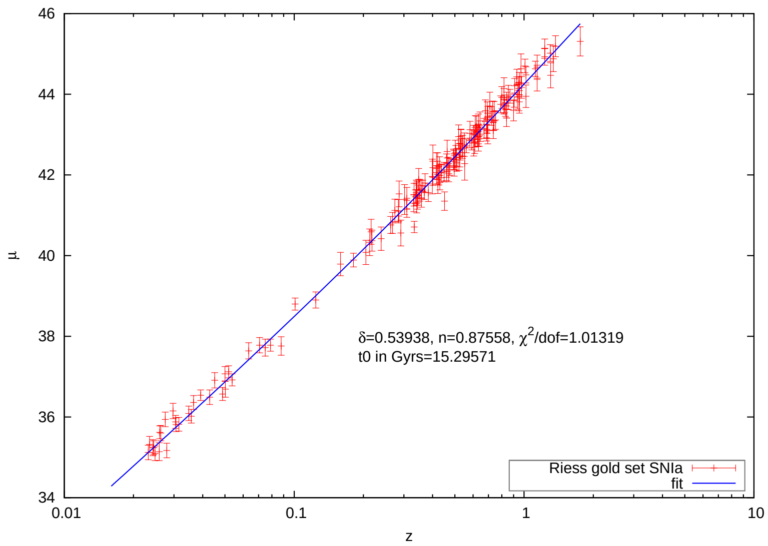
However, due to the imposition of different values of parameters which are given by tachyon constraints (plus anti-Chaplygin gas constraints) the closest singularity in their model appears

– 1 Gyr in future

– and the furthest even 44 Gyr in future.

Despite, they of course can serve as a source of dark energy.

FSF (type III) v. supernovae



We have preliminary found (MPD, Denkiewicz - in progress) that even the type III (Finite Scale Factor) singularity can be closer than this, i.e.

$t_s - t_0 \approx 0.3 Gyrs$ (about 30 times larger than the time to an SFS)

with the choice of parameters to be:

$\ddot{a} > 0$ for $\delta > 0$:

$n = 0.87558$; δ = 0.53938; t_0 = 15.29571 Gyrs

4. Singularities in inhomogeneous cosmology.

Inhomogeneous cosmology - is there a center of the Universe?

- Observations we made are from one point in the Universe and extend only onto the past light cone.
- Even CMB we observe from one point - this proves isotropy, but not necessarily homogeneity (isotropy with respect to any point in the Universe).
- Suppose we have an inhomogeneous model of the Universe with the same (small) number of parameters as a homogeneous dark energy model and they both fit observations very well.
- Could we differentiate between these two models (some tests have been proposed recently but they are not very efficient - see later).
- The simplest models of this type are the models with only one or two parameters which are spherically symmetric (isotropic with respect to only one point).

Center of the Universe?

- There are only two complementary models of the spherically symmetric Universe which can be tested against homogeneous dark energy models.
- These are the inhomogeneous density (dust shells) Lemaître-Tolman-Bondi (LTB) models and inhomogeneous pressure (gradient of pressure shells) Stephani models.
- Apparently, most of the researchers for some reasons investigate the former and only few investigate the latter, though there is no special reason to do so.
- MPD and Hendry (Ap.J. '98) first compared inhomogeneous models (no matter if density or pressure) of the Universe with observational data from supernovae and showed that they can be fitted.
- Despite inhomogeneous density (LTB) models were theoretically explored before (since Lemaître - 1933) only later they were tested observationally (e.g. K. Tomita, prog. Theor. Phys. **106**, 929 (2001); K. Bolejko, astro-ph/0512103).

Complementary models of the spherically symmetric Universe

I will discuss the advantages of the inhomogeneous pressure models and show that they also may fit observations, so that they are a good candidate for explanation of cosmic acceleration by an inhomogeneity.

In order to make a complementary analysis with LTB models the following table proves useful:

	pressure	density
FRW	$p = p(t)$	$\varrho = \varrho(t)$
LTB	$p = p(t)$	$\varrho = \varrho(t, r)$ - nonuniform
Stephani	$p = p(t, r)$ - nonuniform	$\varrho = \varrho(t)$

Living in a "void" against living in an "exotic star"

In the context of dark energy problem there has recently been more interest in LTB models since it could explain the acceleration only due to inhomogeneity - a claim is that

we are living in a spherically symmetric void of density

J. Uzan, R. Clarkson, G.F.R. Ellis (PRL, **100**, 191303 (2008))
R.R. Caldwell and A. Stebbins (PRL, **100**, 191302 (2008))
C. Clarkson, B. Bassett and T. H-Ch. Lu (PRL, **101**, 011301 (2008))
R.A. Sussmann, 0807.1145
K. Bolejko, 0807.2891

I would then suggest that

we are living in a spherically symmetric evolving “exotic star” of variable pressure

Inhomogeneous pressure models - singularities, EOS

- standard Big-Bang singularities $a \to 0$, $\varrho \to 0$, $p \to 0$ are possible (FRW limit)
- **Finite Density (FD)** singularities of pressure appear at **some particular values of the spatial coordinates** x, y, z (or a radial coordinate r, if in a SS model) – in that sense they are spatial singularities – they exist in some regions of the universe in an arbitrary time.
- Π-boundary - a spacelike boundary which divides each negative curvature $k(t) < 0$ section onto the two sheets (the “far sheet” and the “near-sheet”)
- Π-boundary appears whenever $V(t,r) = 1 + (1/4)k(t)[(x - x_0)^2 + \ldots] = 0$
- the Universe behaves asymptotically de Sitter on a Π-boundary ($p = -\varrho$)
- **There is no global equation of state - it changes from place to place (depends on x, y, z or r) and on the hypersurfaces $t =$ const.**

5. Dark energy from spatial singularities of pressure.

Since the acceleration scalar is

$$\dot{u} = -2\frac{a}{c^2}r \quad , \tag{25}$$

with r being the radial coordinate of the model, then

the high pressure region is at $r = 0$ (center of symmetry), while the low (negative) pressure regions are outside the center, so that the particles are

accelerated away from the center

which is a similar effect to that caused by the positive cosmological constant in ΛCDM model.

The difference is that in ΛCDM the pressure is constant everywhere while in Stephani models it depends on the spatial coordinates.

Just very recent stuff: Vanderveld, Flanagan, Wassermann (0904.4319) argue that acceleration at the center of symmetry of LTB models can only appear if the model is not smooth at the center - here we do not have such a problem at all!!! Acceleration appears naturally and everything is smooth at the center.

Inhomogeneous pressure models against supernovae data

In (MPD + Hendry '98) we first compared with Perlmutter P97 (Ap.J. 483, 565 (1997)) data which was in favour of deceleration $(a < 0)$, but the advantage was that inhomogeneous pressure models gave a longer age of the Universe.

According to the current SnIa data (77 supernovae of Riess eta al. for z <0.5) we have the best fit values of inhomogeneity parameter a of the Model I to be (MPD, Denkiewicz - in progress)

$$a = 3645 km^2/s^2 Mpc > 0. \tag{26}$$

Godłowski, Stelmach and Szydłowski (astro-ph/0403534) checked yet another model of the type II which has approximate dust equation of state at the center of symmetry. Their result show that it fits supernovae, Doppler peaks and BBN.

FD singularities versus SFS singularities

- In inhomogeneous pressure models there are Finite Density singularities of pressure.
- In FRW models there are Sudden Future Singularities of pressure.
- However, they are different: FD singularities are spatial (appear somewhere in space) while SFS are temporal (appear in time on one $(t = t_s)$ of the hypersurfaces).
- The question is if SFS may appear in inhomogeneous pressure models, too?
- I have shown (MPD, PLB '05) that this is the case.
- Later, it was also shown by Barrow and Tsagas (CQG **22**, 1563 (2005)) that SFS are possible in Bianchi types homogeneous universes.

FD singularities versus SFS singularities

Such "inhomogeneized" SFS may appear in a general (no symmetry at all) inhomogeneous pressure model which can be shown by inserting the time derivative of the Stephani energy density function and the function $V(t,x,y,z)$ into the expression for the pressure, i.e.,

$$p(t,x,y,z) = -3\frac{\dot{a}^2}{a^2} - 3\frac{k}{a^2} + \frac{\dot{a}}{a}\left[2\frac{\ddot{a}}{a} - 2\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\left(\dot{k}\frac{a}{\dot{a}} - 2k\right)\right]\frac{\left[\frac{V(t,x,y,z)}{a(t)}\right]}{\left[\frac{V(t,x,y,z)}{a(t)}\right]^{\cdot}} . \quad (27)$$

Then, it is easy to notice that SFS $p \to \pm\infty$ appears with (16) for $\ddot{a} \to -\infty$, if $(V/a)/(V/a)^{\cdot}$ is regular and the sign of the pressure depends on the signs of both $\dot{a}/a$ and $(V/a)/(V/a)^{\cdot}$.

In fact, SF singularities appear independently of FD singularities whenever $\ddot{a} \to -\infty$.

6. Conclusions

- Exotic singularities are related to new physical sources of gravity which can serve as the dark energy.
- First example source - phantom - produces an exotic singularity – a Big-Rip in which ($a \to \infty$ and $\varrho \to \infty$) which is different from a Big-Bang/Crunch.
- Investigations of phantom inspired other searches for non-standard singularities (sudden future, generalized sudden future (=Big-Brake), type III (Finite Scale Factor), type IV (Big-Separation), $w-$singularities etc.) which, in fact, are not necessarily the "true" singularities (according to Hawking and Penrose definition), as sources of dark energy.
- Big-Rip which serves as dark energy despite it may happen in 20 Gyr, while weak singularities (of tidal forces and their derivatives) may serve as dark energy if they are quite close in the near future. For example an SFS may even appear in 8.7 Myr with no contradiction with data. A GSFS always appears later. Type III (FSF) is possible in about 0.3 Gyr. Finally, a Big-Brake (which is also an SFS) in tachyon cosmology context is at least 1 Gyr away from now.

conclusions contd.

- Observations from one point in the Universe suggest its isotropy, but not necessarily homogeneity.
- Two specific spherically symmetric models have been proposed: the Lemaître-Tolman-Bondi model (inhomogeneous density) and the Stephani model (inhomogeneous pressure).
- These models have been preliminary checked against astronomical data which shows that the inhomogeneities may play the role of the dark energy (drive acceleration).
- The admission of spherical symmetry would be violating the Copernican Principle. However, the inhomogeneous pressure models have an advantage - they can even model full spacetime inhomogeneity.
- Another advantage is that they admit cosmic acceleration in a natural way since pressure is related to a repulsive force needed for antigravitation.
- An inhomogeneous model with both spatial and temporal pressure singularities can be constructed.